Brombacher
& Associates

Counting

An extract from the NumberSense Mathematics Programme ***Counting, Manipulating Numbers and Problem Solving*** Booklet, which can be downloaded from www.NumberSense.co.za

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Developing a Sense of Number in the Early Grades

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Developing a Sense of Number

Background

As children develop their sense of number there are clearly identifiable stages/milestones: *counting all*, *counting on*, and *breaking down and building up numbers* (*decomposing, rearranging, and recomposing*).

When we observe children at work with numbers – in particular: solving problems with numbers – we can tell what stage of number development they are at.

- If we ask a child to calculate 3 + 5 and we observe that she "makes the 3" and "makes the 5" (using fingers or objects) before she combines the objects and counts all of them to determine that 3 + 5 = 8, then we say that this child is at the *counting all* stage.
- If we observe the child becoming more efficient by "making" only one of the numbers (using fingers or objects) and then counting these objects on from the other number "5: 6, 7, 8" to conclude that: 5 + 3 = 8, then we say that this child is at the *counting on* stage.
- If we observe a child, manipulating numbers to make the calculation easier, for example by saying that 8 + 7 = 8 + 2 + 5 = 10 + 5 = 15, we say that she has reached the *breaking down and building up* stage. What the child has done is to break up one of the numbers: 7 into 2 and 5 which allows her to "complete the 10" by adding the 2 to the 8 and then adding the remaining 5 to the 10 to get 15. We refer to this stage (more formally) as the *decomposing, rearranging and recomposing* stage.

It is expected that all children should reach the *breaking down and building up* stage within age appropriate number ranges.

In the early grades, we support children's development of the number concept through three distinct but interrelated activities:

- **Counting**,
- Manipulating numbers, and
- Solving problems.

Counting

Why is counting important?

Humans have a natural curiosity and tendency to compare:

- *Who has the most? Who has the least?*
- *Who is tallest? Who is shortest?*
- *Who is heaviest? Who is lightest?*
- *Do I have enough? Do I have too much?*
- *Which route is the longest? Which route is the shortest?*

In many situations, we can hold objects next to each other and make direct comparisons. However, when we cannot hold the things we are comparing next to each other, we need a "measuring device". Measuring devices start out informally: fingers, hands, paces and so on. With time these measuring devices become both more efficient and standardised.

As much as we compare objects, we also need to compare the quantity of a collection of objects. How many sheep? How many children? How many days? In order to answer our "how many" questions, we need to count.

Just like the shepherd in the illustration, children must *develop a number system* – they must develop the tools with which to describe quantity/'muchness'. Unlike the shepherd, however, children live in a world with an already well-defined number system. They are born into a society with an existing counting rhyme they do not have to invent their own.

The role of teachers is two-fold. On the one hand the teacher must create opportunities that reveal to children the need/importance/value of counting – counting as a way of describing the 'muchness' of quantities of objects. On the other hand the teacher must introduce children to the socially accepted counting rhyme (*social knowledge* - Piaget).

Counting is not only a tool for answering the "how many" question, it also provides the foundation for the development of number sense, arithmetic and the whole field of mathematics.

Illustration 1 – The Shepherd

Imagine a shepherd in a long forgotten time. Each morning the shepherd goes to the enclosure where the sheep are kept and he releases them for the day to go into the fields and graze. He stands at the entrance to the enclosure with his legs apart. As each sheep passes between his legs he moves a stone from the pile of stones on his left to a pile that he makes on his right.

Counting plays three important roles in the lives of children:

- Counting develops the language of number.
    - Giving meaning to the words often learnt through number songs and rhymes.
- Counting develops a sense of 'muchness' – quantity or numerosity.
    - The idea that 5 is a small number, 50 a bigger number and 500 a much bigger number, and
    - The last number 'counted' tells us the size (measure) of the group.
- Counting provides an early tool for calculating and solving problems.

Guiding the development of counting

Children come to school able to compare. The have an intuitive sense of more and less. They also have a natural interest in quantifying. There are many ways in which this interest and curiosity can be sustained through classroom activities that involve comparing and solving problems through modelling with concrete objects.

Because we want children eventually to develop more efficient (and standardised) ways of working with numbers we introduce them to the number names. We do so by teaching counting rhymes and songs. These are fun for children, especially those with actions. Number songs through their structure (pattern)

In the evening as the sheep return the shepherd reverses the process. For each sheep that passes between his legs he moves a stone from the pile he made in the morning back to the large pile. As the last sheep passes between his legs he moves the last stone back and he knows that all of his sheep are back in the enclosure. Except that there could be days when the last sheep has passed between his legs and there are still stones on the pile – on these days the shepherd knows that some of his sheep are missing – it could be that they got mixed up with his neighbour's sheep. On other days after he has moved the last stone, sheep are still coming into the enclosure – on these days, he knows that his neighbour is missing some sheep.

The shepherd is counting his sheep: for every sheep that leaves the enclosure a stone. There is a one-to-one correspondence between the sheep and the stones.

On those days that the shepherd is missing some sheep he walks over to his neighbour to see if it is possible that his missing sheep are with the neighbour. In order to show the neighbour how many sheep are missing the shepherd carries the pile of remaining stones with him.

Over time the shepherd gets tired of carrying the stones with him as he goes to visit his neighbour. To be sure he also gets tired of moving the stones every day. And so over time he invents a song – a rhyme – that he uses to "count" his sheep: one word per sheep as the sheep leave the enclosure and the same words as the sheep come back every night.

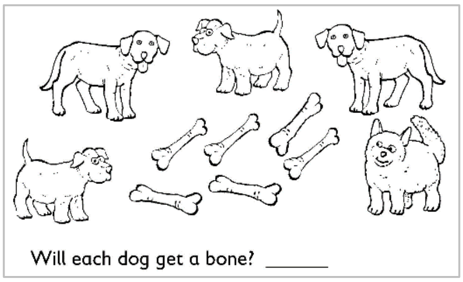
and the story they tell support the memorisation of the number names in sequence. Over time children learn to recite the number names in sequence without the songs that helped them initially. We refer to the reciting of number names in sequence as **rote counting**. Learning to rote count is a necessary early developmental stage in the development of numbers and number sense. There should, however, be no illusion in the mind of the reader, parent and teacher – rote counting (reciting the words of the number song) does not give meaning to the words of the song.

From the very first school day we begin teaching children the number names – we begin with rote counting activities.

Even while children are learning to rote count, we continue to take advantage of their natural curiosity and problem solving abilities by involving them in mathematical tasks such as comparing, sharing, breaking apart and building up. We do so by modelling situations on the mat:

- Which of these teddy bears has more counters? Which teddy bear has less?
- Who can share these counters between these teddy bears so that each teddy bear gets the same number of counters?
- Who can give this teddy bear more counters so that it has the same number of counters as the other teddy bear?
- Who can take some counters away from this teddy bear so that it has the same number of the counters as the other teddy bear?

As children develop confidence with the directly modelled situations on the mat they can begin to solve similar problems present through pictures.



These problems are typically solved by one to one matching: moving counters on the mat and drawing lines on the pictures. Remember how the shepherd in the illustration “counted” by matching stones to sheep? Children are simply counting and solving problems without using numbers (yet!).

Just like the shepherd needed to develop a more efficient way of counting and solving his problem, so the child must develop more efficient ways of counting and solving problems. Children need to begin associating the words of the counting rhyme with the objects they are counting – they need to begin **rational counting**.

As children gain confidence and fluency in rote counting, we introduce the counting of objects—**rational counting**. We do not wait for children to “master” rote counting. The two types of counting build upon each other. Children will typically be able to rote count further than they can count rationally. When they are rote counting they are only concerned with the patterns of the words in the song. When they are counting rational objects they need to focus on associating the words of the song with the objects that they can count.

The shepherd is still counting his sheep: for every sheep that leaves the enclosure a word in the song/rhyme. There is a one-to-one correspondence between the sheep and the words.

On those days that he is missing sheep he still walks over to his neighbour, but now without the stones (he has a more efficient way of knowing how many sheep he is missing). When he gets to his neighbour he recites his rhyme till he reaches the word that corresponds to the number of missing sheep and then they set about looking for the missing sheep.

More time passes and increasingly the two neighbouring shepherds start to use the same rhyme to count the sheep (standardisation) and it is no longer necessary for each shepherd to recite his rhyme to tell the other how many sheep he is missing. The shepherds can now use the unique word in the rhyme that corresponds to the number of missing sheep. The position of the word in the rhyme now suddenly represents not only the sheep that passes through the shepherd’s legs at that moment, but also the collection (number) of sheep that have already passed through his legs.

The story of the shepherd illustrates how counting is a way of measuring quantities – a tool for answering the question “how many?” The story also illustrates how over time as different people work together their ways of counting become more standardised. The counting rhyme 1, 2, 3 … 9, 10, 11 … 99, 100, 101 … that we take for granted has evolved over time.

We expect that:

- Children can rote count in 1s up to 100 and beyond while they are still struggling to count objects up to 30,
- Children can rote count (skip count) in 10s before they can rote counts in 5s and 2s.
- Children can skip count long before they can count rational objects in groups.
- Children can rote count up to 100 and beyond and count rational objects well into the 30s before they can read and write number symbols.
- Children can more easily compare two piles in terms of which pile has more counters in it and which pile has fewer (less) counters in it long before they can compare the “size” of two numbers presented in symbols.

The important point being made is that children can do different things with numbers in different number ranges and teachers need to be sensitive to this. This is what makes the work of the teacher of mathematics in the early grades so incredibly nuanced and challenging. There are many different interrelated concepts being developed and they are developing both independently and yet in relation to each other. The teacher’s role is to know each of the children in her class and to match the range of activities to developmental states of each child.

To illustrate:

- When teachers ask children to trace line patterns they are helping children develop their writing (fine motor) skills long before their “writing has meaning”. And yet, without developing their writing/fine motor skills children will never be able to write.
- When teachers ask children to trace letters and numbers they are helping children develop their skills in making these symbols long before the symbols have meaning.
- When teachers ask children to recite the number rhymes and songs (to rote count) they are helping children develop the sounds of the words long before the words have meaning.
- When teachers ask children to “count the number of objects in a collection”. They rely on the child’s knowledge of the number rhymes, because their counting initially associates the words of the number rhymes with objects in a one-to-one correspondence.
- When teachers ask children who can count rationally in 1s to rote count in 10s, 5s and 2s, it is because they know that the piles of counters being counted is becoming too large to count in 1s and that children will need to develop more efficient ways of counting the large pile: using groups! In order to count in groups the children will need to know (be able to recite) the skip counting songs.
- Etc.

An effective teaching and learning programme brings all of these elements together in an integrated and coordinated way.

In the next part of this chapter we describe a range of developmentally appropriate classroom activities that develop counting. In the Number Sense Workbooks we provide written activities that support the classroom activities.

Before turning our attention to the classroom activities that support counting, there are a few general points that the teacher needs to keep in mind.

A few general comments about counting activities

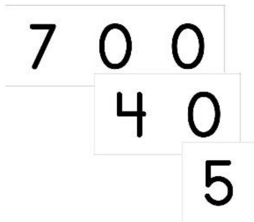
- When introducing rational counting – associating the words of the number rhyme in one-to-one correspondence with the objects being counted – teachers need to:
  - Help children to work systematically.
    - When counting counters this means moving the counters as they are counted. If children do not move the counters from the pile being counted, they run the risk of counting the same counter again. This becomes all the more important as the number of counters increases.
    - When counting objects in a written context this means crossing out the objects that have already been counted.
  - Choose the objects to be counted carefully. Counters should be easy to handle and should ideally all look the same. Large white beans (kidney beans) are very useful as counters. Large buttons and even bottle tops also make good counters. Building blocks are not necessarily good objects as children get distracted and want to join them together and build structures. Each teacher needs a large collection of counters in her class.

- Answering the question "How many?" represents a developmental shift for children. At first when we point to a pile of counters and ask a child to count the counters they will do so associating the words of the rhyme in one-to-one correspondence with the objects until there are no counters left (or until they start to make mistakes). If we then ask the child "So, how many counters are there in the pile?" they will often start counting the pile again: "one, two, three …". This is because they associate the question "How many?" with the process of counting. To answer "five" to the question "How many?" means that the child must recognise "five" as both the "name" of the last counter: "one, two, three, four, five" and as a description of the number of counters in the collection. The point is that teachers need to be aware of the conceptual demand that the question represents and to be patient with children as they reach of the point of seeing the double meaning of the "five" in this example. Lots and lots of counting practice will support children in making the shift.

- Writing the number symbol associated with the number of counters counted needs support. Just because children can make the symbol does not mean that they associate the symbol with the number.
    - In the lower number range (up to around 120) a number chart can be a great help. When the child has counted the objects and concluded that there are 7 counters, the teacher will hold up the number chart and ask: “Can you show me where the seven is?” If the child points to the seven, the teacher will say: “That is correct” and ask the child to write the number in her book. If the child does not know what the seven looks like, the teacher will say: “Let us look for it” she will ask the child to point to the one and then count pointing to the numbers on the number chart in sequence, counting: “one, two, three, … seven” and stop on the seven. The teacher then says: “That is what the seven looks like. Please write it down.” Not only does this help the child to learn the symbols associated with the number names, but it also provides the child with a way of ‘looking for’ number symbols when the child does not have the teacher to help her.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140

    - In the higher number ranges (from the 50s to the 1 000s) Flard cards can be both helpful in teaching children how to write numbers, while at the same time helping them to develop a better sense of how numbers break up (decompose). For example, helping children to think of 745 as 700 + 40 + 5.



    - Two cautionary comments:
        - Number charts are very helpful in developing the social knowledge of number symbols. Number charts are not helpful in helping children perform calculations. For calculations number lines are far more helpful. The problem with number charts is that while it is possible to establish that 17 + 5 = 22, the 22 is “to the left of” 17 – the relative size is not at all obvious. Teachers, however, persist in teaching children to calculate on number charts because it makes sense to them, forgetting that children do not necessarily have the same experience of the teacher.
        - As much as decomposing numbers into so-called hundreds, tens and units (as in: 745 = 7H + 4T + 5U) makes complete sense to the teacher of the early grade child, it does not make the same sense to the child. Children in the early grades learn more about the structure and meaning of number by decomposing 745 as 700 + 40 + 5 and this is supported through the effective use of Flard cards.

- Comparing numbers as in "Which number is larger 14 or 36?" is not, initially, as easy for the child as it is for the adult teacher. Comparing two piles of counters (that are obviously different in size) in terms of answering the question "Which pile has more counters?" is much easier for children. To answer the question "Which number is larger 14 or 36?" requires that a child has a sense of the 'muchness' of 14 and of the 'muchness' of 36. This 'muchness' is developed through rational counting activities involving estimation, counting and comparing estimates with the actual numbers. The key point is, once again, that patience is needed in helping children develop their ability to compare number symbols in terms of the size of the number. The development of this skill is supported through counting activities.

Counting activities for the classroom

Rote counting activities

Rote counting activities are appropriate for children who do not know the number names (i.e. the counting vocabulary). Typically children who start school do not know the number (counting) words or vocabulary. Before children can start counting rationally they must first learn the number (counting) words. Children learn the words best by learning the words in number 'rhymes' or 'songs'. As they learn the various number 'rhymes' and 'songs' they must become increasingly aware of the underlying patterns. In particular, they should realise that by combining the pattern 1; 2; 3; ... 9 with the pattern 10; 20; 30; … 90 they can say the number rhyme from 1 to 99.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	…						30
31	…								40

In due course when they also learn the number pattern 100; 200; 300; … 900 they can expand the rhyme to go from 1 to 999.

Although rote counting plays an important role in developing the child's number vocabulary (i.e. the number names) children who can rote count do not necessarily associate meaning with the words.

In order for the numbers to have meaning to children, they need to engage in **rational counting** activities.

While it is tempting to engage the children in whole-class rote counting/chanting activities very few children benefit from this. In general, children hide in the group and the activity is nothing more than a way of keeping the class busy.

Rote counting activity 1: Counting in ones repeating after the teacher

In this activity the children repeat the number sequence after the teacher. This is necessary until the children can do so by themselves.

<u>Activity</u>

- The teacher says the sequence: "one, two , three, four, five"
- The children repeat the sequence after the teacher.
    - First, they do so as the small group sitting on the mat that the teachers is working with and/or as a whole class.
    - Next, they can do so as members of a small group (with or without the supervision of the teacher). All of the children in the small group repeat the sequence together.
    - Finally, they do so as individuals in a small group. As one individual recites the sequence so the others in the group listen carefully to see if they agree with the individual.
- As the children gain confidence so the teacher extends the sequence to:
    - one, two, three, … ten;
    - one, two, three, … fifteen;
    - one, two, three, … twenty; and so on.

As children gain confidence in their rote counting abilities so the teacher can expand the rote counting activities to include an increasing number range. The number "up to which children count" should increase as their confidence increases.

Rote counting activity 2: Number songs and rhymes

In addition to reciting number words after the teacher, children also benefit from learning the number words as part of a song. If the song is accompanied by actions the children become physically involved in the counting activity and this adds to their enjoyment of the activity. The purpose of the songs is to increase children's familiarity with the number words (in sequence).

Activity

- Teach children to sing number songs and rhymes including the actions that accompany the words.
- Songs can be repeated frequently and added into the classroom routine. For example, some teachers teach children songs involving numbers that they sing as they enter or leave the classroom.

There are many number songs such as:

One two buckle my shoe
Three, four, knock at the door
Five, six, pick up sticks
Seven, eight, lay them straight
Nine, ten, a big fat hen
Eleven, twelve, dig and delve
Thirteen, fourteen, maids a-courting
Fifteen, sixteen, maids in the kitchen
Seventeen, eighteen, maids in waiting
Nineteen, twenty, my plate's empty

Rote counting activity 3: Counting in steps repeating after the teacher

As children gain confidence with the counting in 1s sequence, the teacher can add rhythm to the counting. This is so that children start to develop a greater sense of the many skip counting "patterns" within the number sequence. For example, when counting in 1s the teacher can draw attention to the multiples of five by having the children clap as they say each multiple: 1, 2, 3, 4, **5 (clap)**, 6, 7, 8, 9, **10 (clap)**, 11, 12, 13, 14, **15 (clap)**. The children will not recognise these numbers as multiples of five; they are simply responding to the rhythm in the number pattern and are getting ready to count in fives.

Activity

- The teacher says the sequence: "one, two, three, four, five … twenty" and she claps when she says the numbers five, ten, fifteen and twenty.
- The children repeat the sequence after the teacher clapping as the teacher did.
  - First, they do so as the small group sitting on the mat that the teacher is working with and/or as a whole class.

  - Next, they do so as members of a small group (with or without the supervision of the teacher) – with all of the children in the small group repeating the sequence and clapping together.
  - Next, they do so as individuals in a small group. As one individual recites the sequence and claps for the multiples of five so the others in the group listen carefully to see if they agree with the individual.
- As the children gain confidence the teacher repeats the activity above saying the words that are not multiples of five softer and softer until only the multiples of five can be heard.
- Children should now be able to rote count in fives
- As the children gain confidence the teacher should extend the activity to:
  - Involve increasingly larger number ranges
  - Include counting in fives, tens, twos and hundreds
  - Counting forwards and backwards first in ones then in fives, tens and twos.

Rational counting activities

As children develop their knowledge of the number names (i.e. their rote counting abilities) they become ready to start rational counting. Rational counting is the act of counting physical objects (things). Rational counting involves answering the question “How many?” For example, “How many children?”, “How many cars?”, “How many blocks?” etc.

Children need a lot of help as they start rational counting. They need to associate the number names of the number rhymes with the objects they are counting in a one-to-one manner. At first children will simply say the number words in sequence as they point to objects at random; they won’t necessarily know when to stop counting and may even count some objects more than once.

The teacher’s role is to help children organise the objects that they are counting. At first children must touch the objects as they count them. It is best if they move the objects from one place to another as they are counted.

The objects used as counters are also important. Counters should be easy to handle and all look the same. Large white beans (kidney beans) are very useful as counters. Bottle tops and large buttons also make good counters. Building blocks are not good objects because children get distracted by wanting to build structures with them. Each teacher needs a large collection of counters in her class.

As children gain confidence with rational counting, they also need to start recognising and writing number symbols. In general, children will be able to count further than they can read and write the number symbols.

Different children will be able to count objects (rational counting) correctly up to different amounts. It is best to allow each child to count as far as he/she can. Teachers should avoid deciding on a target for the whole class before the counting activities.

Rational counting activity 1: Using number cards

The activities described here are appropriate for very young children who have not yet learnt to rote count as well as for those who are starting to count rationally.

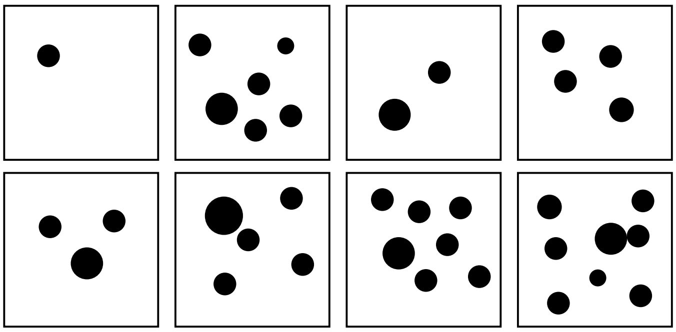
These activities also help develop visual discrimination and encourage reasoning about more, less and equal.

Materials needed (a print master has been supplied as an appendix to this teacher guide)

- A set of cards with dots
    - Cards should have a wide range of number of dots in different positions and of different sizes.
    - Cards with the same number of dots must have the dots in a variety of orientations.
- A set of cards with number symbols
- A set of cards with number names

Activity

The teacher should work with small groups of children at a time. The tasks described below are listed from easiest to hardest. The teacher needs to choose the task that is most appropriate to the child's developmental state.



8	**5**	**4**	**2**
1	**7**	**3**	**6**

one	**seven**	**four**	**five**
two	**six**	**eight**	**three**

- Task 1: The teacher shows the children two cards with dots on them and asks "Which card has the most dots on it?" or "Which card has the least number of dots on it?"
    - The teacher should start with two cards that have a very different number of dots on them. This is because the more similar numbers of dots on the two cards, the harder this activity is for the children. The teacher can progress to progress to cards with a more similar number of dots once they feel the child is ready.
- Task 2: The teacher gives the children a few cards and asks them to arrange the cards from the card with the least number of dots to the card with the greatest number of dots on it. Cards with the same number of dots should be placed on top of each other.
- Task 3: The teacher gives the children two sets of cards. One set of cards has dots and the other set has number symbols. The child must match the cards with dots to the corresponding cards with number symbol.
    - This task is not suitable until children know the number symbols (these are developed in the activities that follow).
    - The more cards that the children are given, the harder the task becomes.
- Task 4: The teacher gives the children three sets of cards. One set has dots, one set has number symbols and one set has number names. The children must match the cards with dots to the corresponding cards with number symbols and number names.

Rational counting activity 2: Counting small piles of counters individually

This is the first rational counting activity that children should engage in.

Materials needed

- A large set of counters. See the earlier note about the kinds of objects that make good counters.
- A large number chart with the numbers 1 to 120 or more (a print master has been supplied as an appendix to this teacher guide).
- A writing book for each child to write in.
- A pencil for each child to write with.

Activity

- The teacher arranges small groups of children (8 to at most 10) in a circle on the mat. The teacher should be a part of the circle, so that he/she can make eye contact with each child.
- The teacher places a pile of counters in front of each child. The size of the pile is determined by the teacher's knowledge of how far the children can count. Each child gets a pile of counters that is a little larger than the number to which the child can count.
- Each child in the group gets a turn to count the counters in their pile of counters. It is important that:
  - The child touches each counter as they count it. He/she should move each counter from one place to another as they count it.
  - The other children in the group listen carefully to the child who is counting and help him/her if he/she gets stuck.
  - The teacher makes a note of how far each child can count before making a mistake.
- As children gain confidence with the task the teacher can expand the activity by asking each child to point to the number that they counted to on a number chart. As a further extension the children can be asked to write the number that they counted in their writing books. These extensions are important because they contribute to children learning to read and write number symbols and names.
  - When the child has finished counting, the teacher holds the number chart up and asks the child "Can you show me where the number ** is on this chart?"
    - If the child points to the correct number the teachers points to the number and says "That is correct. That is what the number ** looks like. Everybody write down the number." All of the children write the number in their writing book.
    - If the child does not know what the number looks like, or points to the incorrect number, the teacher says "Let us look for the number." The teacher then helps the child to point to the numbers on the chart one by one while counting: "one, two, three …" and so on until the child gets to the number. At this point the teacher says "That is correct. That is what the number ** looks like. Everybody

write down the number." All of the children write the number in their writing book.

- As children gain more confidence with the task the teacher can begin to ask the question "How many counters were there in your pile?"
    - This question will not be obvious to children at first and many will start counting the pile again: "one, two, three ..." The reason for this is that children associate the counting process with the question "How many?"
    - The question also marks an important moment in the development of the child's understanding of number. The number "9" suddenly represents not only the name of the last counter counted, but also the number of counters in the pile. Teachers need to be sensitive to this developmental difficulty and must be patient with children as they work out this dual meaning of the number.

<u>NOTE</u>: This activity is only really suitable for children who are counting up to at most 30 counters. Once children can count rationally beyond 30 they need to move to rational counting activity 4.

Rational counting activity 3: Counting out a given number of counters

This activity is an extension of rational counting activity 2 but is listed separately because of the important difference in nature of the task.

In order for children to be able to solve problems involving numbers they need to have a sense of the meaning of numbers. Children who are at the 'counting all' stage of number development need to be able to 'make a number' in order to solve problems involving the number. Practically, this means that children need to be able to make a pile with a given number of counters in it. This is why this activity is important.

<u>Materials needed</u>

- A large set of counters. See earlier note about the kinds of objects that make good counters.

<u>Activity</u>

- The teacher arranges small groups of children (8 to at most 10) in a circle on the mat with the teacher a part of the circle, so that she can make eye contact with each child.
- The teacher places a large pile of counters in the middle of the group and one by one asks each child to make a pile with a given number of counters in it. Each child will make a pile with a different number of counters to the other children. The size of the pile that a child makes is determined by their developmental state. The teacher says to the child "Please make a pile with *11* counters in it."
    - The child must then count out the correct number of counters from the pile in the middle of the group and place these in front of them.

  - The other children must watch as the child counts out his/her pile and must offer help if they think the child has made a mistake.
- As children gain confidence in reading and writing number symbols, the teacher can change the instruction to "Please make a pile with so many counters in it" pointing to the number on her number chart and/or writing the number on a small chalkboard.

Rational counting activity 4: Estimating and counting in 1s taking turns

This activity is appropriate for children who are able to individually count objects of 30 or more. This activity also contributes to children's shift from 'counting all' to 'counting on'.

Materials needed

- A large set of counters. See earlier note about the kinds of objects that make good counters.
- A large number chart with the numbers 1 to 120 or more (a print master has been supplied as an appendix to this teacher guide).
- A writing book for each child to write in as well as a pencil for each child to write with.

Activity

- The teacher arranges small groups of children (8 to at most 10) in a circle on the mat with the teacher a part of the circle, so that she can make eye contact with each child.
- The teacher places a pile of counters in the middle of the group. She then asks the children to look at the pile of counters and to estimate how many there are. The teacher tells the children to write down their estimate in their writing books.
  - Note: the number of counters in the pile is related to the number range in which the group of children is counting. Typically the pile should be a little larger than the number up to which the children in the group can count. It is, therefore, important that the teacher knows the developmental state of each child in the group.
- After each child has written down their estimate the teacher asks each child to tell the group their estimate. After each child has told the group their estimate the teacher can ask questions such as:
  - "Whose estimate is the largest?"
  - "Whose estimate is the smallest?"
  - "Whose estimates are bigger than 20?"
  - "Whose estimates are smaller than 50?"
- The teacher now asks the first child to start counting the counters and stops the child after he/she has counted for a while (see comments below). After stopping the child the teacher asks all of the children in the group to write down in their books the number reached by the child who was counting. If the children struggle to write down the number the teacher can again use the number chart as she did in rational counting activity 2.

- After each child has written the number and the teacher has checked that they have done so correctly, the teacher asks the next child to continue counting the pile of counters. The child continues from where the previous child stopped and moves the counters that he/she counts to the pile of counted counters. The teacher asks the child stop after he/she has counted for a while (see comments below) and asks all of the children in the group to write down the number reached by the child who was counting in their books.
- The activity continues in this way until all of the counters have been counted.
- When all of the counters have been counted and the children have all written down the number of counters in the pile, the teacher can ask questions such as:
    - "Whose estimate was closest to the number of counters in the pile?"
    - "Whose estimate was furthest from the number of counters in the pile?"
    - "How far was your estimate from the actual number?"

<u>A few comments about the activity</u>:

- Each child who gets a turn to count should count at least 14 to 20 counters so that they are forced to count through at least one or two _9 to _0 transitions. The reason for this is that children typically struggle with the decade transitions but not with the numbers inside a decade once they get there. For example
    - The first child counts past 9 to 10 and beyond and past 19 to 20 and beyond before the teacher stops her at about 26,
    - The next child then counts past 29 to 30 and beyond and also past 39 to 40 and beyond before the teacher stops him at 44. And so on.
- Make sure that each child counts through at least one decade transition. For example:
    - If there are 8 children in the group and there are about 65 counters in the pile, then let each child count through two decade transitions and to let the fourth child start at the beginning again. This is better than letting each child only count a few counters but not getting an opportunity to count through the decade transitions.
- If a child makes an estimate that is far too low it is a good opportunity to address it when counting the counters. For example:
    - There are 120 counters in the pile and one of the children has estimated that there are 25 counters. The teacher, then, may stop the first child who is counting at 25 and speak to the child who estimated 25 saying "You estimated 25. Are 25 or more than 25 counters in the whole pile? Do you want to change your estimate?" This will help children to develop a stronger sense of the amounts associated with different numbers.
- It is important that the teacher plans to count in this way well beyond one hundred as soon as possible. Grade 1 children, in the second half of the year, should be able to reach this point. There are at least two reasons:

    - Firstly, it is unnatural to have an "upper limit" when counting. Counting beyond 100 assists with the development of a child's sense of 'muchness' and exposes children to the pattern that underpins the structure of the counting numbers.
    - Secondly, by stopping at 100, children develop a sense that the world stops at 100 and when asked to count on from 100 say things like "98, 99, 100, 200, 300 ..." or some variation.
- As children start manipulating numbers (see the later section) teachers can enrich the counting activity by asking questions such as:
    - "How many more counters do we need to get to 60? Let's check."
    - "How many more counters do we need to get to 100? Let's check."

NOTE: This activity is only really suitable for children who are counting up to at most 200 counters. Once children can count rationally beyond 200 they need to move to more efficient counting strategies – namely, counting in groups (see rational counting activity 6).

Rational counting activity 5: Counting in groups – counting body parts

As children gain confidence with counting in 1s they should be encouraged to start counting in groups. The human body provides some natural groups. The body has two eyes, two ears, two hands and two feet. The body has 10 fingers, 10 toes, two groups of five fingers on each hand and two groups of five toes on each foot. The groups found on the human body are at the heart of the base ten number system. It is interesting to notice that the body does not have groups of 3 or groups of 4. Counting in groups of 3 and groups of 4 is not a natural thing to do and does not contribute to the development of children's sense of number.

Activity

- After completing rational counting activity 4 for the day, the teacher asks "How many ears (or eyes, hands or feet) does Sipho have? How many ears does Mary have? How many ears does Piet have? Who can count all of the ears in the group?"
- The teacher asks a volunteer to walk around counting the ears of all the children in the group. The volunteer must stand up and walk around the group touching each of the ears that he/she counts.
    - The first volunteer will typically count "one, two, three ... sixteen"
- The teacher asks the question "Who can count the ears more quickly?" She then chooses a volunteer to count the ears of all the children in the group. The volunteer must stand up and walk around the group touching each of the ears that he/she counts.
    - The second volunteer will typically repeat what the first volunteer did, but do so more quickly.

- The teacher repeats the question: "How many ears (or eyes, hands or feet) does Sipho have? How many ears does Mary have? How many ears does Piet have? Who can use this fact to count all of the ears in the group as quickly as possible?"
    - The next volunteer will typically touch two ears at a time and count "two, four, six ... sixteen".
- At this point the teacher can ask other volunteers to repeat the counting. The process of counting in groups has been initiated. Children need a lot of practice with this activity.

A few comments about the activity:

- This rational counting activity is not an independent activity that needs a great deal of time. It is an add-on to rational counting activity 4 which should be used when children are ready to start counting more efficiently. It would probably be appropriate to start using this activity in the second half of Grade 1.
- The developmental sequence for counting in groups in this way is to first count in twos (ears, eyes, hand and feet), then in tens (fingers and toes) and only then in fives (fingers on a hand or toes on a foot).
    - When counting fingers (in tens and fives) it is important that each child in the group place their hands on the mat so that all of their fingers are visible and that the child counting the fingers touches the groups of fingers aware that they are counting a group and not single items using a "different song".

Rational counting activity 6: Counting in groups – counting large piles of counters efficiently

As children start to count with confidence into the hundreds and after they have been introduced to counting body parts in groups they are ready to start counting large piles of counters efficiently. This is an activity that will typically be used from the last few months in Grade 1 right through to the end of Grade 3 with the only difference being the size of the pile of counters being counted. This activity assumes that rational counting activities 4 and 5 are familiar to the children.

Materials needed

- A large set of counters. See earlier note about the kinds of objects that make good counters.
- Flard cards (a print master for the Flard cards has been supplied as an appendix to this teacher guide).
- A writing book for each child to write in as well as a pencil for each child to write with.
- Containers that can be used to store the counters counted on one day when the children will count further on the next day.

Activity

- The teacher arranges small groups of children (8 to at most 10) in a circle on the mat with the teacher a part of the circle, so that she can make eye contact with each child.
- The teacher places a large pile of counters in the middle of the group. She asks the children to look at the pile of counters and to estimate how many there are. The teacher tells the children to write down their estimate in their writing books.
    - Note: The number of counters in the pile is determined by the developmental state of the children in the group. For Grade 1s there might be between 70 and 130 counters. For Grade 3s there could be more than 400 counters.
- When each child has written down their estimate the teacher goes around the group and asks each child to tell the group their estimate. After each child has told the group their estimate the teacher can ask questions such as:
    - "Whose estimate is the largest?"
    - "Whose estimate is the smallest?"
- The teacher says to the children "If we counted this pile of counters in ones we would be busy all day. How could we count the pile more quickly?" The children will suggest counting the pile in twos or tens or fives (developed in rational counting activity 5). The teacher helps the children agree on the group size for counting the pile of counters.
    - Note: The teacher's response to the suggestions of the children is determined by the number of counters in the pile. For piles up to 70 or 80 counting in twos makes sense and the teacher may agree to counting in twos. For piles larger than 70 or 80, the teacher will suggest that the pile is counted in groups of 10 or 5. The teacher may also suggest that a pile is counted in groups of 10 or 5 if the she wants to practice counting in 10s or 5s.
- **If the pile is to be counted in twos** the teacher asks the children in the group to take turns counting in twos. The children should move two counters at a time, as happened when counting in 1s in rational counting activity 4. As with rational counting activity 4, the teacher stops each child after they have counted between 14 and 20 counters, the children in the group all write down the number and the next child continues until the pile has been counted.
- **If the pile is to be counted in tens or fives** the teacher asks the children in the group to take counters from the large pile and to make piles of 10 (or 5) in front of themselves. Each of the children gets involved and continues in making piles until all of the counters in the middle pile have been used up. It should be expected that there will be a few left over counters and these are left or put back in the middle of the mat.
- Once the large pile has been arranged in small piles of 10 (or five) counters the first child touches her piles counting one by one counting "ten, twenty, thirty, ... fifty" until all her piles are counted and the next child continues counting "sixty, seventy, eighty, ... one hundred and ten" until all his piles are counted. Continue in this way until all the counters have been counted.

- The children in the group use the Flard cards to make the total and write it into their writing books (see separate note on working with Flard cards).
- When all of the counters have been counted and the children have all written down the number of counters, the teacher can ask questions such as:
    - "Whose estimate was closest to the number of counters in the pile?"
    - "Whose estimate was furthest from the number of counters in the pile?"
    - "How far was your estimate from the actual number?"
- Once children have gained confidence in the activity and **if the pile was counted in tens or fives** the teacher may now encourage the children to rearrange their piles into more organised arrangements and then to count the counters again in increasingly larger groups:
    - If the pile was reorganised into groups of 10 then these groups of ten can be arranged:
        - In rows of two groups of ten so that the counters can be counted in 20s
        - In rows of five groups of ten so that the counters can be counted in 50s
        - In rows of ten groups of ten so that the counters can be counted in 100s
            - For a pile of 348 counters the teacher may help the children to see that three rows of ten piles of counters leads to 300, another four piles of counters leads to 40 and there are 8 loose counters: 348 = 300 + 40 + 8.
    - If the pile was reorganised into groups of 5 then these groups of five can be arranged:
        - In rows of two groups of five so that the counters can be counted in 10s
        - In rows of four groups of five so that the counters can be counted in 20s
        - In rows of five groups of five so that the counters can be counted in 25s
        - Etc.

<u>A few comments about the activity:</u>

- After organising the counters into rows and logical larger groups, the teacher can draw children's attention to the fact that the groups sometimes allow for counting in different ways. For example, if the counters are grouped in fives and there are there are five piles of counters in four rows, then the collection can either be counted as 25 + 25 + 25 + 25 = 100 or as 20 + 20 + 20 + 20 + 20 = 100. In other words, 4 groups of five 5s is the same as five groups of four 5s. So, 4 fives = 5 fours. By drawing attention to these relationships teachers are drawing children's attention to the commutative property of multiplication, namely that $a \times b = b \times a$.
- Counting large piles of counters in groups is important for several reasons:
    - Counting large numbers is important because it helps children to develop a sense of quantity ('muchness'). For example, unless children have worked with 748 they have no sense that 748 is a large number, that 74 is much less and that 8 is really small by comparison.
    - Counting large piles in groups makes the point that in mathematics our goal is to be efficient. 748 counters could be counted in 1s and moved them from one group to

another, but that could result in errors *and* it would take a very long time. Using groups is more efficient than counting in 1s both in terms of time and in terms of accuracy.

  - Counting large piles of counters in groups that link to the base 10 number system expose children to the structure of numbers. It draws attention to the fact that 748 = 700 + 40 + 8 and that 9 748 = 9 000 + 700 + 40 + 8 etc.

- Although children can count several hundred counters very quickly in groups (as described), teachers may be concerned about starting at 1 again every day. In order to get to larger numbers more quickly, the teacher may keep the counters that were counted on one day and count on from that pile the next day. For example, if 368 counters counted on a particular day the teacher then places them in a container writing 368 on the side. The next day when the children count again the teacher can make a pile with the 368 next to another pile of counters. The children then count on from 368 to determine the number of counters in the two piles combined.

NOTE: This activity is suitable for children who can:

- Count beyond 100 in 1s with confidence, and
- Who have learnt the skip counting patterns through rote counting activities.

There is no upper limit to the number of counters that children can count in this way. By Grade 3 children should be counting several hundred counters in this way. However, it is not very important that children count counters beyond 1 000.

Manipulating Number (in full version)

Problem Solving (in full version)