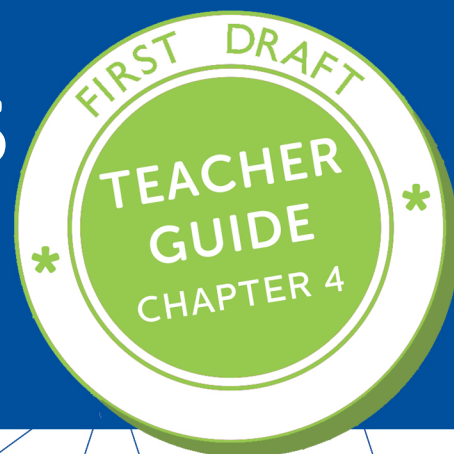# PATTERNS, FUNCTIONS & ALGEBRA

Brombacher & Associates

PATTERNS AND PATTERNING

Background

Mathematics is, among other things, the study of patterns.

Patterns allow us to describe a changing world. Patterns and structures are fundamental to mathematics and it is for this reason that patterning is critical to the development of mathematical proficiency.

Patterns are relationships between variables with some form of regularity and rules that describe them. From a young age we need to nurture children's ability to recognise and describe variation, and to make predictions about:

- what will happen under certain conditions; and
- what conditions are needed for specific outcomes to occur.

There is evidence that children's ability to recognise, describe, and extend patterns is predictive of their future success in mathematics. It is for this reason that the study of patterns and patterning is a critical component of mathematics in the early years.

The challenge of school mathematics is not so much to 'teach patterns and patterning' as a topic but, to support children to notice patterns, to describe them, and to make predictions based on those patterns. In addition, children need to be supported to develop the vocabulary with which to describe patterns.

Patterns in the Grades R to 3 and 4

In the early school years, patterns reveal the structure of mathematics and patterns provide a tool for learning mathematics.

- **Counting involves patterns**
  Counting to 100 involves combining two related patterns, 1 to 9 and 10 to 90.
  If children recognise counting to 100 as combining these two patterns, they will not have to remember the numbers 1 to 100 as a sequence of 100 unrelated names.

- **Proficiency and efficiency in calculating is developed by noticing patterns**
  If children know that, for example, 5 + 2 = 7, then, by recognising the pattern(s), they will realise that:
  25 + 2 = 27; 45 + 2 = 47; 145 + 2 = 147; and 500 + 200 = 700 and so on.

Being aware of the commutative property of both addition and multiplication, and recognising the interrelatedness between addition and subtraction and between multiplication and division, supports children in realising that:

If 5 + 3 = 8, then it follows that: 3 + 5 = 8; 8 − 5 = 3; 8 − 3 = 5 and so on.

and, since: 4 × 5 = 20, it follows that 5 × 4 = 20; 20 ÷ 4 = 5; 20 ÷ 5 = 4 and so on.

The ability to double and halve supports multiplication.

Knowing that 5 is one half of 10 supports children to multiply by 5:

Since 24 × 10 = 240 and 5 is one half of 10 it follows that: 24 × 5 is equal to half of 240 … 120.

- **Geometry:**

  By making a pattern with shapes (tessellating) it is possible for children to use the tessellation to convince themselves of the properties of geometric shapes, e.g. squares, rectangles, parallelograms, and so on.
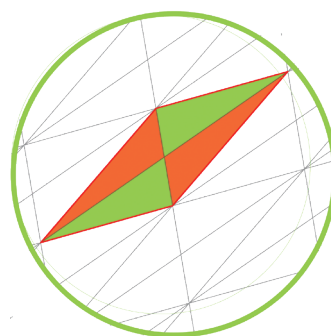
Patterns in Grades 3 to 7 and beyond

The focus of patterns and patterning in this phase of schooling shifts to the study of patterns as patterns. That is, the focus shifts to developing algebraic thinking/reasoning.

Algebraic thinking/reasoning involves generalising, representing, and analysing patterns and relationships to understand and solve mathematical problems.
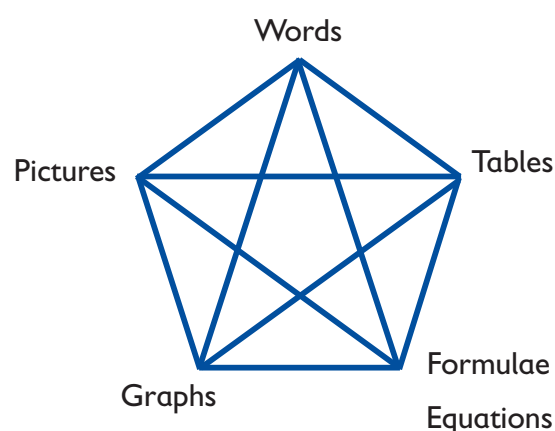

Figure 1: Different possible representations of mathematical relationships

Patterns and relationships can be represented in different ways (*Figure 1*)

Pictures

Concrete and visual representations (pictures) are key to understanding the relationship(s) between the variables in a situation. Early in their mathematical journey, children benefit by drawing or building patterns using resources such as matchsticks, bottle tops and so on. The process of drawing or building patterns help children "feel" the relationships between the variables in the situation (Piaget's physical knowledge).

However, drawing pictures or building physical objects is not limited to young children. Mathematicians also draw pictures or build objects to understand relationships.

Words

Describing patterns and relationships verbally helps children to explain the relationships between the variables in detail. This enables them to provide explanations that might not be apparent in the concrete or pictorial relationships.

Tables

Tables organise the variables in the patterns and relationship into rows and columns, showing the relationship between the variables through discrete data points. Tables also provide easy access to the corresponding values of the variables in patterns and relationships.

Formulae/equations

Formulae and equations reduce patterns and relationships to symbolic representations that can be understood and manipulated in the absence of the pattern that gave rise to them. Equations are particularly helpful in solving problems related to the situation that they summarise.

Graphs

Graphs provide a visual representation of patterns and relationships that reveal the interplay between the variables in the situation in ways that the other representations are unable to do.

Formulae/equations and graphs can be organised into groups (families) based on the form of the equation and the corresponding shape of the graph. For each group (family) there is a general equation with variables that change according to the situation being represented. At primary school, we are mostly concerned with so-called linear relationships, direct proportion relationships (which are a special case of linear relationship), and developing an awareness of the existence of non-linear relationships.

Interrelationship between the representations

Exploring and describing the interrelationships between these representations is crucial to developing a deep understanding of patterns and relationships. The different representations complement each other, providing a more comprehensive understanding of the patterns and relationships being investigated.
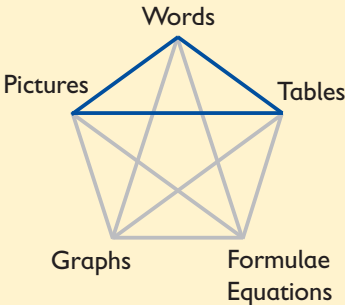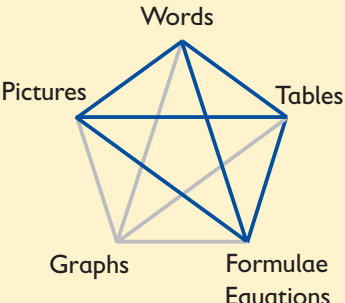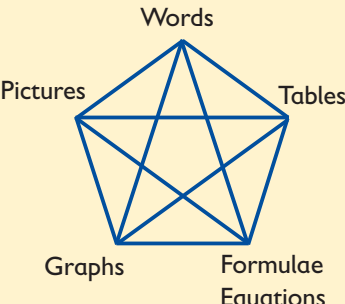
Patterns & patterning

	Grade R	Grade 1	Grade 2	Grade 3
Early grade foundations for algebraic reasoning	Extending and completing geometric border type and numerical patterns.	Counting in groups (1s, 2s, 5s, 10s, 20s and 100s), extending and completing numerical and geometric border type patterns.	Counting in groups, extending and completing numerical and geometric border type patterns. Recording information in tables.	Counting in groups, extending and completing numerical (including fractions) and geometric border type patterns. Recording information in tables. Using tables to solve problems.

Words, Pictures, Tables, Graphs, Formulae Equations

Words, Pictures, Tables, Graphs, Formulae Equations

Words, Pictures, Tables, Graphs, Formulae Equations

Figure 2: Developmental trajectory for patterns and patterning in primary school

Grade 4	Grade 5	Grade 6	Grade 7
Extending picture patterns; describing patterns in words; recording characteristics of patterns in tables; and, describing the relationship between the characteristics of the picture patterns and the corresponding tables.			
Direct proportion and linear patterns.	Direct proportion, linear, and non-linear patterns.		
	Extending picture patterns; describing patterns in words; recording characteristics of patterns in tables; describing input and output characteristics of the table (first by means of flow diagrams and later using equations); and, describing the relationships between the characteristics of the picture patterns, tables, flow diagrams and equations.		
			Use and create formulae to describe situations and solve equations to answer questions about situations.
	Direct proportion, linear, and non-linear patterns (quadratic, hyperbolic and exponential).		
	Read data off and interpret non-algebraic graphs of situations (e.g. temperature and rainfall patterns etc.).	Draw graphs for the situations, tables of values and formulae/ equations developed to date. Describe the relationships between the characteristics of the picture patterns, tables, flow diagrams, equations and graphs.	

Pictures show the situation that gives rise to the pattern, while words are needed to describe it. Tables record specific instances of the pattern/relationship that are not visible in the pictures or words. Formulae summarise the relationship in a way that enables the making of predictions of an outcome under certain conditions (using a formula) and the conditions that are needed for desired outcomes (solving equations). Graphs make the relationship(s) between the variables visible in ways that the pictures, words, tables, and formulae cannot.

By using these different representations in their study of patterns and relationships, children develop a more robust understanding of the relationship(s) between the variables in patterns and relationships.

Figure 2 summarises the developmental trajectory for patterns and patterning in the NumberSense Mathematics Programme from Grade R to Grade 7 in terms of both the nature of the tasks and the representations used at each stage of the journey.

What comes next?
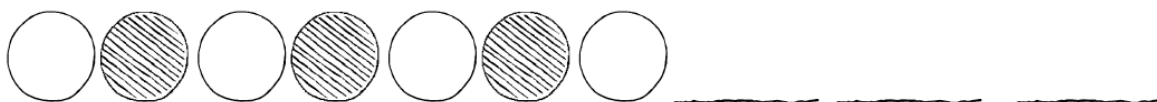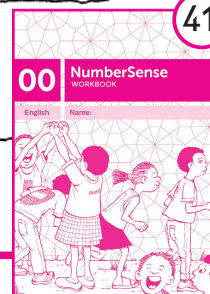
What comes next?

What comes next?
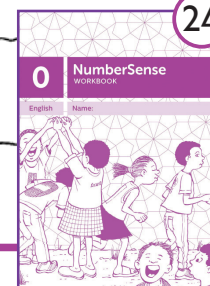
This is a table of numbers.

Girls		1	2	3	4	5
Ears		2	4	6	8	10

Can you see how it works?

2. Complete the table.

Dogs		1	2	3	4	5	6
Ears		2	4				

20

2 NumberSense
COMPREHENSIVE WORKBOOK
English   Name

1. Complete.

15   18   21

70   85   90   95

106   107   108

39

3 NumberSense
COMPREHENSIVE WORKBOOK
English   Name
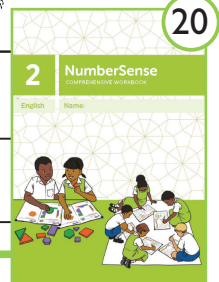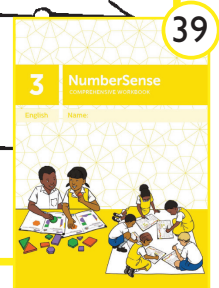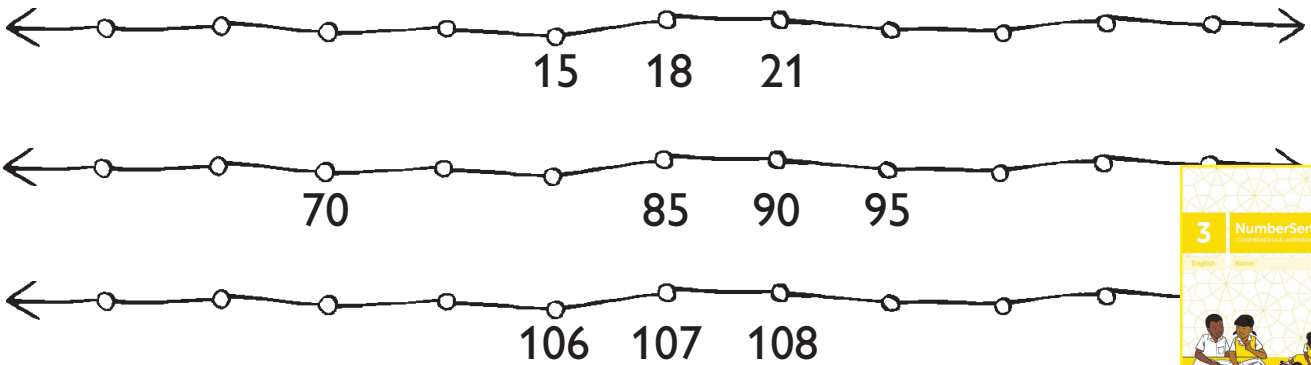
2. Complete the tables.

Children	1	2	3	4	5	8	10	15
Hands	2							

Children							
Hands	20	30	34	40	48	50	60

16

4 NumberSense
COMPREHENSIVE WORKBOOK
English   Name
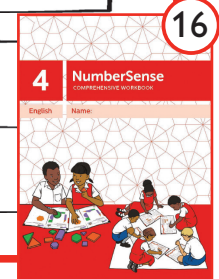
4. Complete the tables.

5c coins	1	2	3	4	5	6	10	11
Cents	5	10						

10c coins	1	2	3	4				
Cents		20	30			50	60	70
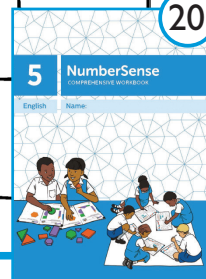
6. Extend and colour the patterns.
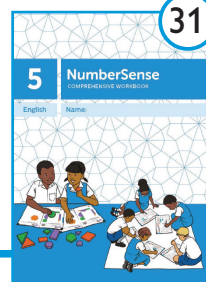
3. Complete the table and the number sentences.

Children	1	2	5	10	20	30	31	32
Hands	2	4						

Children	2							
Hands	4	20	40	44	48	52	56	60

$5 \times 2 =$ _____ $7 \times 2 =$ _____

$6 \times 2 =$ _____ $8 \times 2 =$ _____
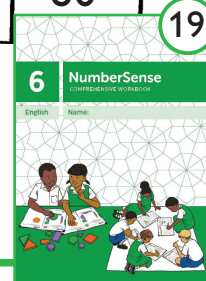
2. Jan saves money in his money box. He saves the same amount of money each month. Complete the table.

Months	0	1	2	3	4	5	6	7
Rand	0	6	12					

- How much money does he save every month?
- How much money will there be in his box after 10 months?

3. Ben's granny gave him a money box with R10 in it. He saves R4 every month. Complete the table.

Months	0	1	2	3	4	5	6	7
Rand	10	14	8					

How much money will there be in his box after 10 months?

2. Complete.

50 ⟶
65 ⟶
80 ⟶ ×2 ⟶ ____   ____ ⟶ ×2 ⟶ 100
95 ⟶   ____   ____ ⟶ 120
105 ⟶   ____   ____ ⟶ 140
               ____   ____ ⟶ 150
               ____   ____ ⟶ 180
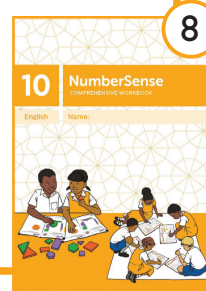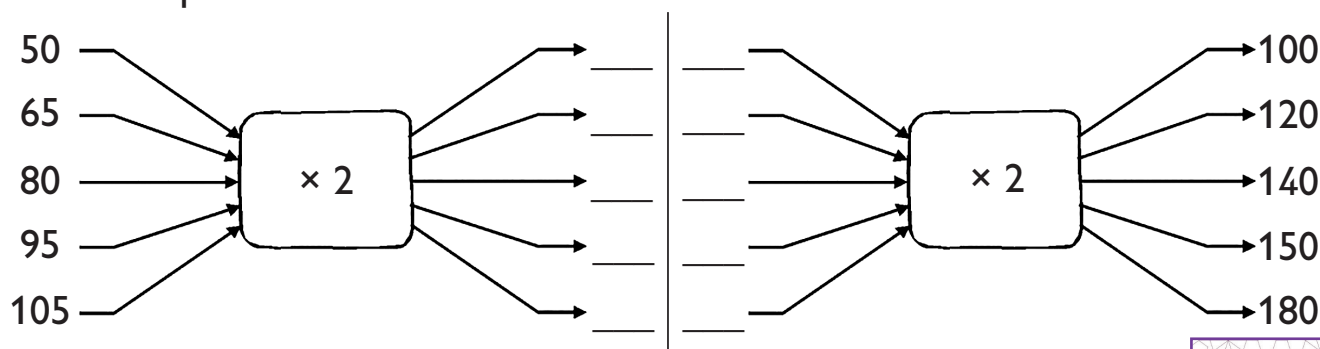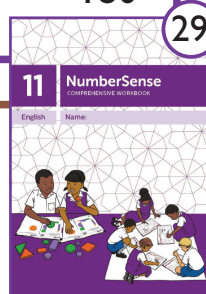
2. Complete each row of numbers.

36 ; 43 ; 50 ; ___ ; ___ ; ___

53 ; 61 ; 69 ; ___ ; ___ ; ___

147 ; 151 ; 155 ; ___ ; ___ ; ___

- What will the eighth number in each row be?
  Row 1 ___   Row 2 ___   Row 3 ___
- What will the tenth number in each row be?
  Row 1 ___   Row 2 ___   Row 3 ___
- What will the fifteenth number in each row be?
  Row 1 ___   Row 2 ___   Row 3 ___
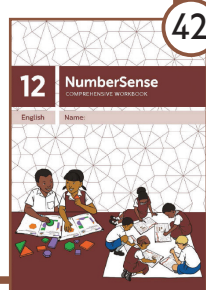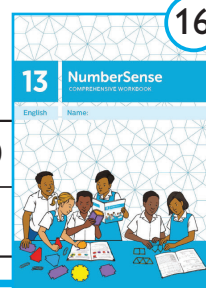
1. Yusuf makes pictures with squares like this. The first 3 pictures make a pattern.
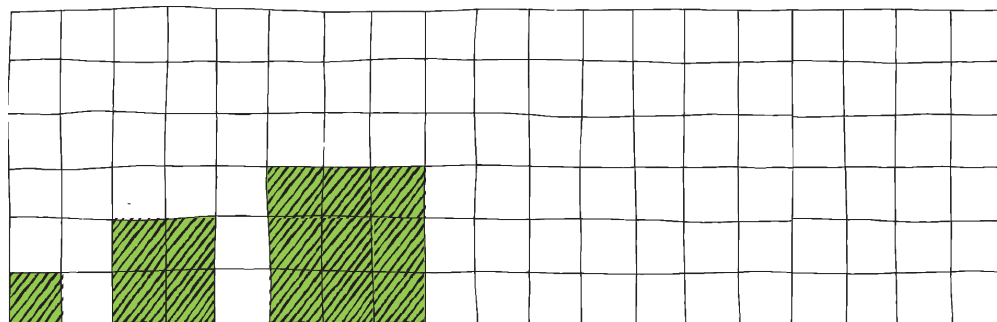
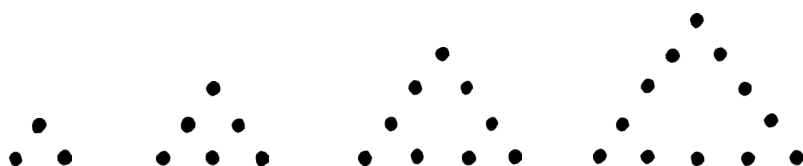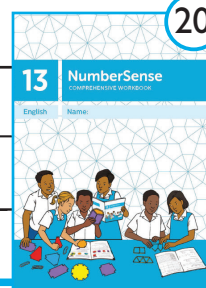Picture 1   Picture 2   Picture 3   Picture 4   Picture 5

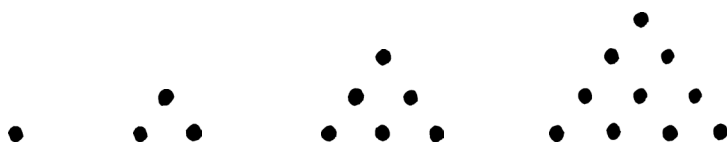a. Draw the fourth and fifth pictures in the pattern.

b. Complete the table for the number of squares he needs for each picture.

Picture number	1	2	3	4	5	6	7	8	9	10
Number of squares	1	4								

16

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1. Beth makes pictures with dots like this. The first 4 pictures make a pattern.

Picture 1   Picture 2   Picture 3   Picture 4   Picture 5   Picture 6

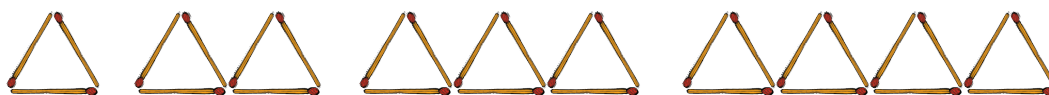a. Draw the fifth and sixth pictures in the pattern.

b. Complete the table.

Picture number	1	2	3	4	5	6	7	8	9
Number of dots	3								

c. How many dots does she need for picture 12?

20

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1. Bongi makes pictures with dots like this. The first 4 pictures make a pattern.

Picture 1   Picture 2   Picture 3   Picture 4   Picture 5   Picture 6

a. Draw the fifth and sixth pictures in the pattern.

b. Complete the table.

Picture number	1	2	3	4	5	6	7	8	9	1
Number of dots	1	3	6	10						

c. How many dots will she need for picture 12?
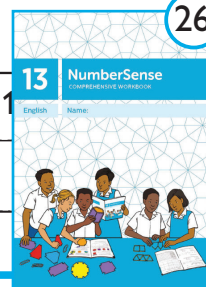
26

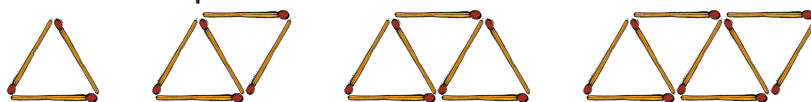1. Jan makes pictures with matches like this. The first four pictures make a pattern.

    a. Draw the fifth picture in the pattern.

    b. Complete the table for the number of matches in each picture.

Picture number	1	2	3	4	5	6	12	13	20
Number of matches	3	6							

2. Sara makes pictures with matches like this. The first four pictures make a pattern.

    a. Draw the fifth picture in the pattern.

    b. Complete the table for the number of matches in each picture.

Picture number	1	2	3	4	5	6	12	13	20
Number of matches	3	5							

c.

Sara is calculating the number of matches needed for each picture.

For picture 1, I need 1 times 2 plus 1, and 1 × 2 + 1 is 3.
For picture 2, I need 2 times 2 plus 1, and 2 × 2 + 1 is 5.

Use Sara's method to calculate the number of matches needed.

Picture 1   1 × 2 + 1 = 3

Picture 2   2 × 2 + 1 = 5

Picture 3   _____ + 1 = _____

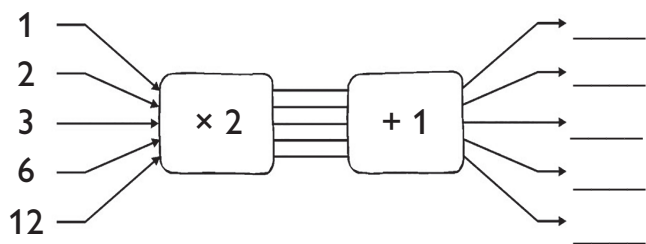Picture 4   _____ + 1 = _____

Picture 6   6 × 2 + 1 = _____
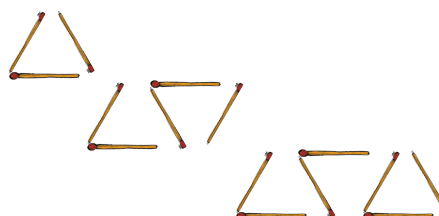
d. How many matches will Sara need to make a picture with 45 triangles?

1. Ben makes pictures with matches like this. The first 4 pictures make a pattern.
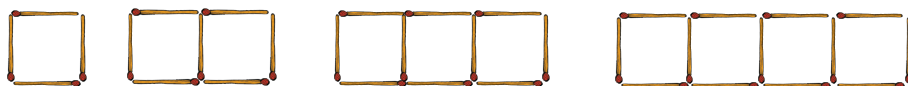
a. Draw the fifth picture.

b. Complete the table for the number of matches in each picture.

Picture number	1	2	3	4	5	6	12	13	14	20
Number of matches	4	7								

c.

Ben is calculating the number of matches needed for each picture.

For picture 1, I need 1 times 3 plus 1, and 1 × 3 + 1 is 4.
For picture 2, I need 2 times 3 plus 1, and 2 × 3 + 1 is 7.

Use Ben's method to calculate the number of matches needed.

Picture 1 $1 × 3 + 1 = 4$

Picture 2 $2 × 3 + 1 = 7$

Picture 3   _____ + 1 = _____

Picture 4   _____ + 1 = _____

Picture 6 $6 × 3 + 1 =$ _____

Picture 12   _____ + 1 = _____

d. Ben has 91 matches. How many squares can he make?

e. How many squares can he make if he has 302 matches? How many matches will he have left over?
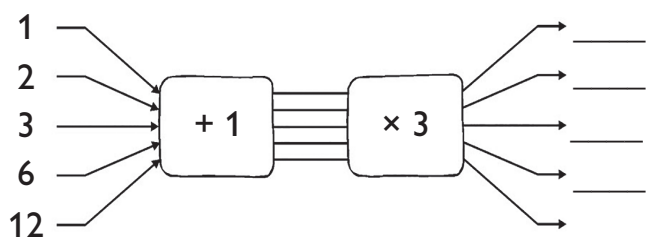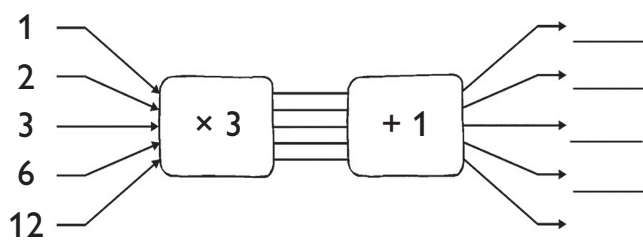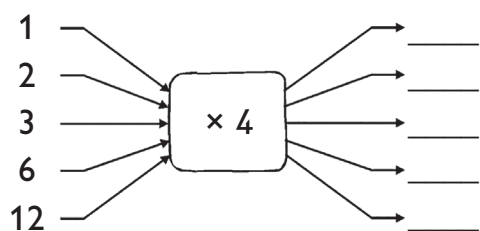
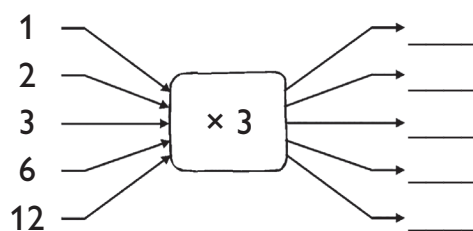1. a. Complete the following flow diagrams.

A

1
2
3   × 2   + 1   _____
6   _____
12   _____

B

1
2
3   + 1   × 2   _____
6   _____
12   _____

C

1
2
3   + 1   × 3   _____
6   _____
12   _____

D

1
2
3   × 3   + 1   _____
6   _____
12   _____

E

1
2
3   × 4   _____
6   _____
12   _____

F

1
2
3   × 3   _____
6   _____
12   _____
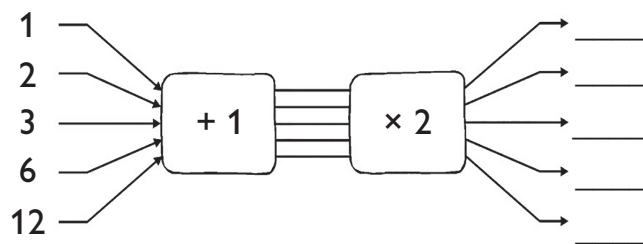
b. Which of the flow diagrams summarise the methods used by Jan, Suzi and Themba? Write down the letter only.
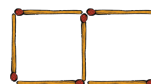
- Jan's pattern with triangles on page 3. _____
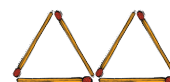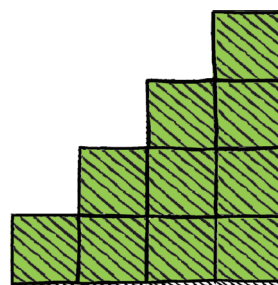
- Sara's pattern with triangles on page 3. _____
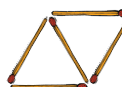
- Ben's pattern with squares on page 6. _____

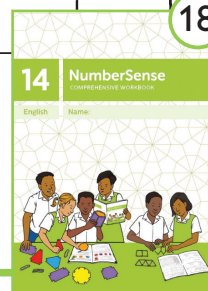1. A builder uses blocks to build staircases. The staircase in the sketch needs **10** blocks and is 4 steps high.

   Complete the table to show the number of blocks needed for different heights of staircases.

Number of steps	1	2	3	4	5	6	7	8	9	10
Number of blocks	1	3	6	10						

   How many blocks will he need for a staircase which is:

   a. **12** steps high?

   b. **15** steps high?

1. The Khumalo family is going camping for the holidays. Mrs Khumalo booked them into a camping site. They have to pay R50 to get into the site and then R75 per night that they stay there.

   Complete the following table.

Number of nights	1	2	3			6	7	8	9
Cost (R)				350	425				

   a. How much will it cost them to stay 4 nights?

   b. How much will it cost them to stay 10 nights?

   c. Vusi calculates the cost for 14 nights by doubling the cost of 7 nights. Is he correct? Test Vusi's plan by using the values in the table or extending your table.

   d. Complete.

   1
   2
   3   × 75   + 50
   4
   5
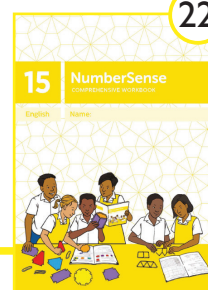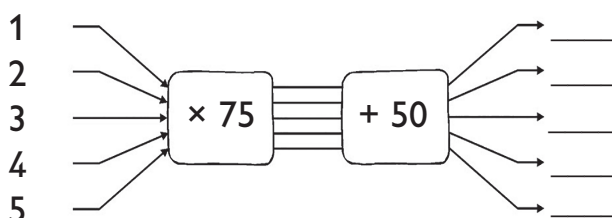
1. Lebo makes pictures with black dots. The first four pictures make a pattern.

Picture 1   Picture 2   Picture 3   Picture 4   Picture 5   Picture 6
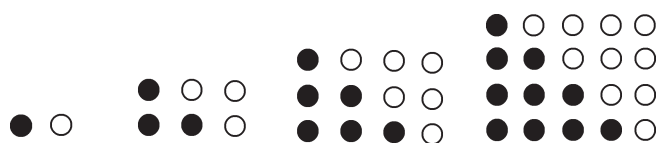
  a. Draw the fifth and sixth pictures in the pattern.
  b. Complete the table.

Picture number	1	2	3	4	5	6	7	8	10
Number of dots	2	6	12	20					

  c. How many dots will there be in picture 12?

2. Anwar makes pictures with black and white dots. The first four pictures make a pattern.

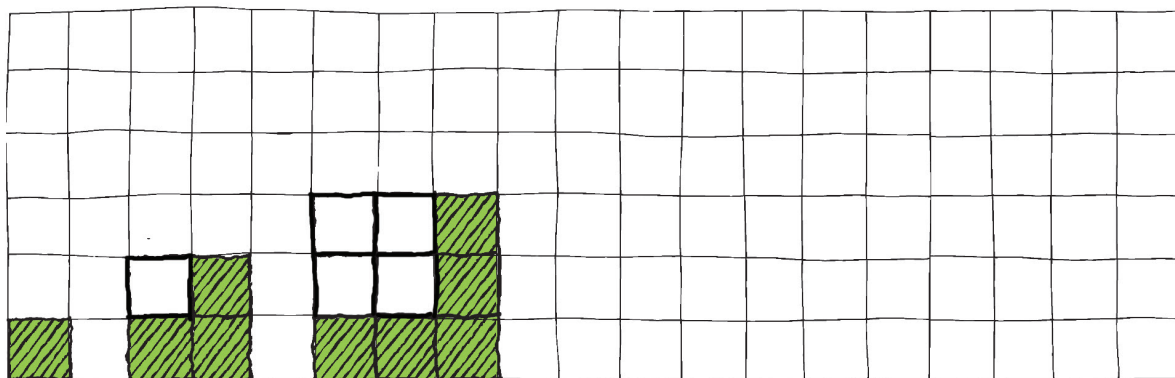Picture 1   Picture 2   Picture 3   Picture 4   Picture 5   Picture 6

  a. Draw the fifth and sixth picture in the pattern.
  b. Complete the table.

Picture number	1	2	3	4	5	6	7	8	10
No. of black dots	1	3							
No. of white dots	1	3							
Total no. of dots	2								

  c. How many black dots will there be in picture 12?

3. Describe how Anwar's pictures help you to determine the number of white dots and the number of black dots in his pattern.

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14 NumberSense
COMPREHENSIVE WORKBOOK
English   Name:

1. Piet makes pictures with green and white tiles like this. The first 3 pictures make a pattern.
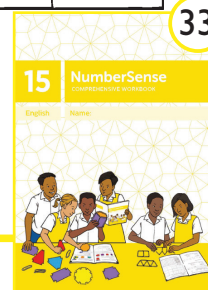

Picture 1   Picture 2   Picture 3   Picture 4   Picture 5

   a. Draw the fourth and fifth pictures in the pattern.

   b. Complete.

Picture number	1	2	3	4	5	6	7	8	9
No. of green tiles	1								
No. of white tiles	0								

   c. How many green tiles will there be in the 10th picture?

   d. How many white tiles will there be in the 10th picture?

33

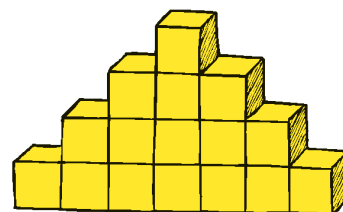1. Grace makes arrangements with building blocks like this. The first 4 arrangements make a pattern.
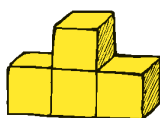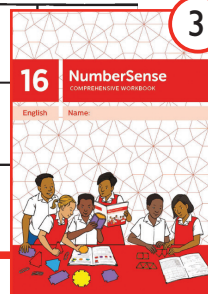

Arrangement 1   Arrangement 2   Arrangement 3   Arrangement 4
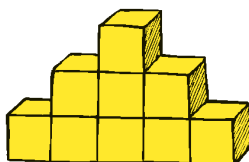
   a. Complete the table.

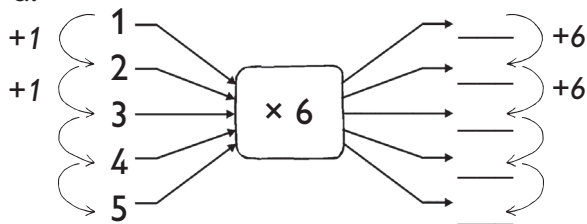| Arrangement number	1	2	3	4	5	6	7	8
Number of blocks	1	4						

   b. How many blocks will she need for arrangement number 12?

3

1. Complete.

a.

		+1	+1	+1	+1
Input	1	2	3	4	5
Output	6				

+6 __ __ __

b.
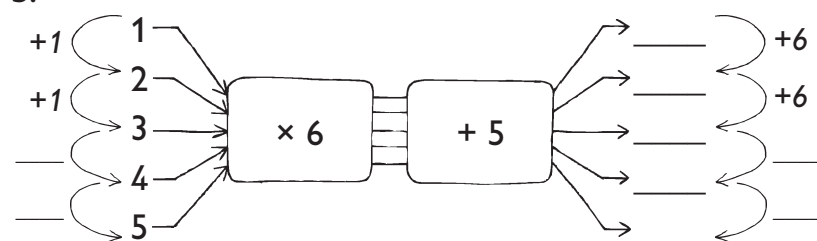
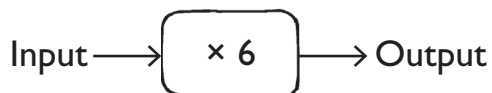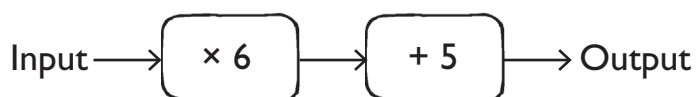|		+1	+1	+1	+1
Input	1	2	3	4	5
Output	11				

+6 __ __ __

Have you noticed that flow diagrams describe the relationship between pairs of input and output numbers, and that tables are used to record pairs of input and output numbers?

The relationship between input and output numbers can be summarised by a general flow diagram.
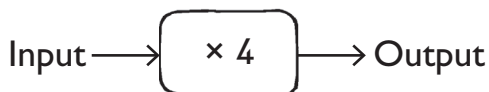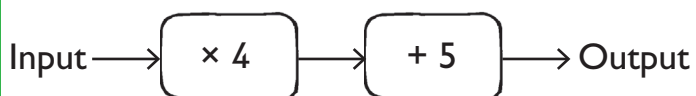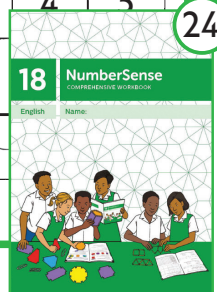
For question 1a: Input ⟶ [× 6] ⟶ Output

For question 1b: Input ⟶ [× 6] ⟶ [+ 5] ⟶ Output

2. Complete.

Input ⟶ [× 4] ⟶ Output

		+1	+1	+1	+1
Input	1	2	3	4	5
Output					

Input ⟶ [× 4] ⟶ [+ 5] ⟶ Output

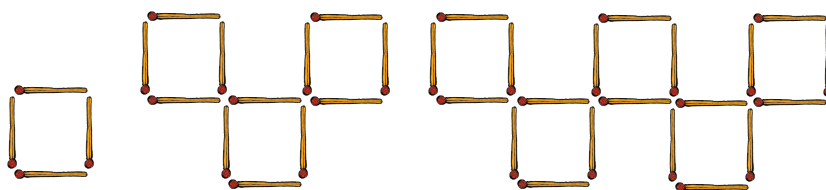|		+1	+1	+1	+1
Input	1	2	3	4	5
Output					

24

18 NumberSense
COMPREHENSIVE WORKBOOK

English   Name:

1. Grace makes pictures with matches like this. The first three pictures make a pattern.
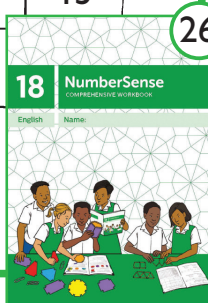
Picture 1   Picture 2   Picture 3   Picture 4

a. Draw the fourth picture in the pattern.

b. Complete the table.

Picture number	1	2	3	4	5	6	8	10	15
Number of matches	4	12							

c. How many matches will she need for picture 20?

26

18 NumberSense
    COMPREHENSIVE WORKBOOK
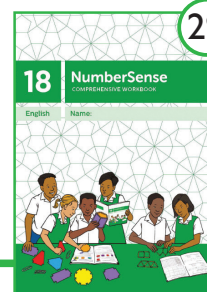English   Name:

1. The length of a coil spring increases when mass pieces are attached to it.

a. Complete the table for the length of the spring as different numbers of the same weight are attached to it.
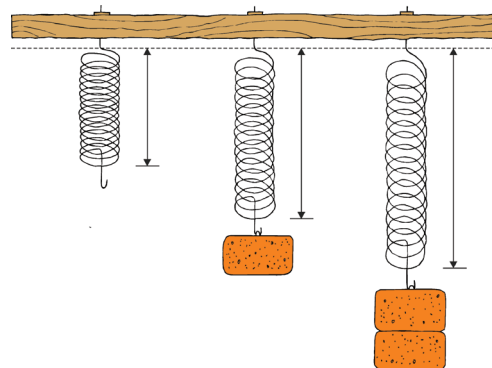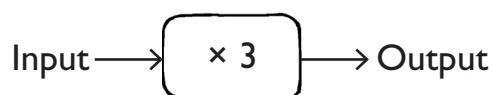
No. of weights	0	1	2	3	4	5	6	7	8
Length of spring (mm)			58		66	70			

b. By how much does the length of the spring increase if the load increases by one weight piece?

c. What is the length of the spring when there are no weights attached to it?

d. Write a flow diagram to show how you can calculate the length of the spring if you know how many weights are attached to it.
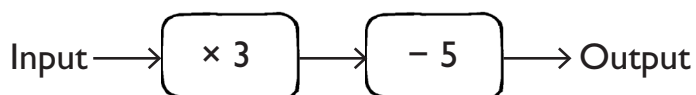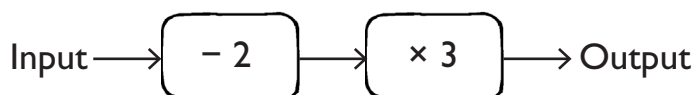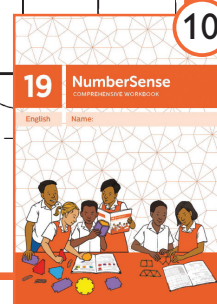
29

18 NumberSense
    COMPREHENSIVE WORKBOOK
English   Name:

1. a. Complete.

Input → × 3 → Output

		+1	+1	+1	+1
Input	4	5	6	7	8
Output					

Input → × 3 → − 5 → Output

		+1	+1	+1	+1
Input	4	5	6	7	8
Output					

Input → − 2 → × 3 → Output

		+1	+1	+1	+1
Input	4	5	6	7	8
Output					

10

19 NumberSense
COMPREHENSIVE WORKBOOK
English   Name:

b. For each of the tables, determine by how much the output number increases if the input number increases by 1.

2. Alan and Liam make pictures with tiles like this ▱. The first three pictures make a pattern. The patterns start with three tiles in picture 1.

Alan

Picture 1   Picture 2   Picture 3   Picture 4

Liam

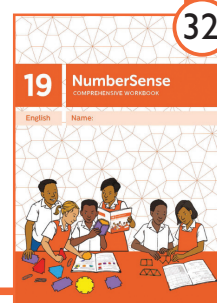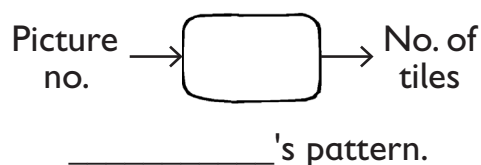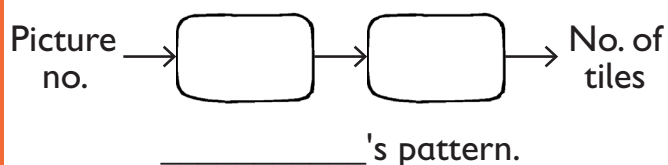Picture 1   Picture 2   Picture 3   Picture 4   Picture 5

a. Draw the fourth and fifth pictures in each pattern.
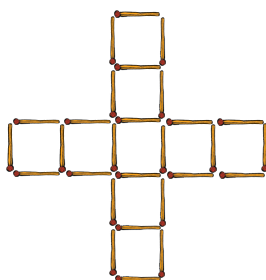
b. Select and complete the appropriate flow diagram for the number of tiles in Alan and Liam's patterns. Explain.
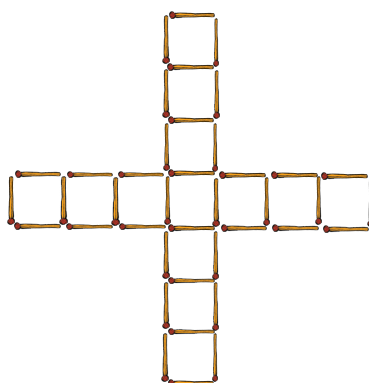
32

19 NumberSense
COMPREHENSIVE WORKBOOK
English   Name:

Picture no. → ☐ → ☐ → No. of tiles

_____'s pattern.

Picture no. → ☐ → No. of tiles

_____'s pattern.

1. Neba makes pictures with matches like this. The first three pictures make a pattern.
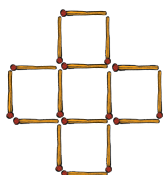
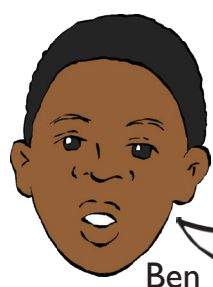Picture 1   Picture 2   Picture 3   Picture 4

a. Draw the fourth picture in the pattern.

b. Complete the table.

Picture number	1	2	3	4	5	6	7	8	10	12
Number of matches	16	28	40							

Here are 4 different ways of determining the number of matches in picture 30.

Ben: *We need 12 more matches for each new cross. So, I keep on adding 12 to the number of matches until I get to picture 30.*

Casey: *I know how many matches we used for picture 10. I multiply that number by 3 because 3 × 10 = 30.*
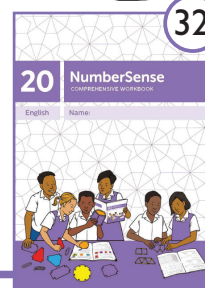
Vusi: *There are 16 matches in picture 1. I add 12 matches for each new picture. For picture 30, I add 29 × 12 to 16.*

Sindi: *I see the cross has 4 matches in the middle. Each arm grows by 3 matches each time. So, I add 4 × 3 = 12 matches every time. The pattern is:*
12 × 1 + 4;   12 × 2 + 4;
12 × 3 + 4, and for picture 30
12 × 30 + 4 = 364

c. Who will get the correct number of matches?

Test all 4 methods on the values in your table.

32

Ayanda draws pictures with dots like this. The first three pictures make a pattern.

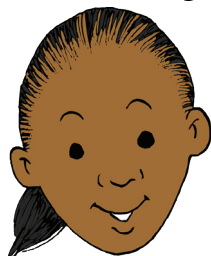| Picture 1 | Picture 2 | Picture 3 | Picture 4 | Picture 5 |

1.   Draw the fourth and fifth pictures in the pattern.

2.

The class wants to work out how many dots there will be in picture 100. Look at some of their ways and answer the questions.

By now I know that when numbers in the top row of the table increase by one and the numbers in the second row increase by the same amount then I can make a flow diagram to calculate the answer.

Sally

a. Use Sally's thinking to complete the table and make a flow diagram.

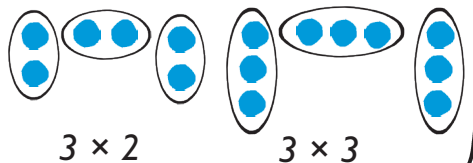| Picture no.	1	2	3	4	5
No. of dots	6	9			

Pic no. → ⬚ → ⬚ → No. of dots

b. Use the flow diagram to calculate the number of dots in picture 100.

3.

I just look at the pictures and notice three equal groups.

Abdul   *3 × 2*   *3 × 3*

Use Abdul's method to calculate the number of dots in picture 100.

Picture 1:   3 × 2 = 6

Picture 2:   3 × 3 = 9

Picture 3:   3 × ___ = ___

Picture 4:   3 × ___ = ___

Picture 100:   3 × ___ = ___

4.

Xolile

I break up the picture into a top bit and two side bits.
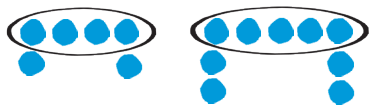
4 + 2 × 1   5 + 2 × 2

Use Xolile's method to calculate the number of dots in picture 100.

Picture 1:   4 + 2 × 1 = 6
Picture 2:   5 + 2 × 2 = 9
Picture 3:   ___ + 2 × ___ = ___
Picture 4:   ___ + 2 × ___ = ___
Picture 100: ___ + 2 × ___ = ___

5.

Zoliswa

I fill up the rectangle and compensate.

2 × 4 − 1 × 2   3 × 5 − 2 × 3

Use Zoliswa's method to calculate the number of dots in picture 100.

Picture 1:   2 × 4 − 1 × 2 = 6
Picture 2:   3 × 5 − 2 × 3 = 9
Picture 3:   ___ × ___ − ___ × ___ = ___
Picture 4:   ___ × ___ − ___ × ___ = ___
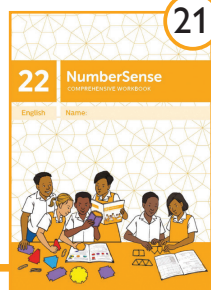Picture 100: ___ × ___ − ___ × ___ = ___

Four different ways of calculating and writing their thinking! Different calculations, which result in the same outcome, are called equal or equivalent.

6.   The different methods for picture 1 are written next to each other. Complete for picture 100.

	Sally	Abdul	Xolile	Zoliswa
Picture 1:	3 × 1 + 3	3 × 2	4 + 2 × 1	2 × 4 − 1 × 2
Picture 100:				

7.   Elsie saw the expressions in each child's method for picture 4 and calculated the number of dots. Explain why Elsie did not always get the expected result.

	Sally	Abdul	Xolile	Zoliswa
Picture 4:	3 × 4 + 3	3 × 5 = 15	7 + 2 × 4	5 × 7 − 4 × 5
	= 12 + 3		= 9 × 4	= 35 − 4 × 5
	= 15		= 36	= 31 × 5
				= 155

Ayanda draws pictures with dots and crosses like this. The first three pictures make a pattern.

| Picture 1 | Picture 2 | Picture 3 | Picture 4 | Picture 5 |

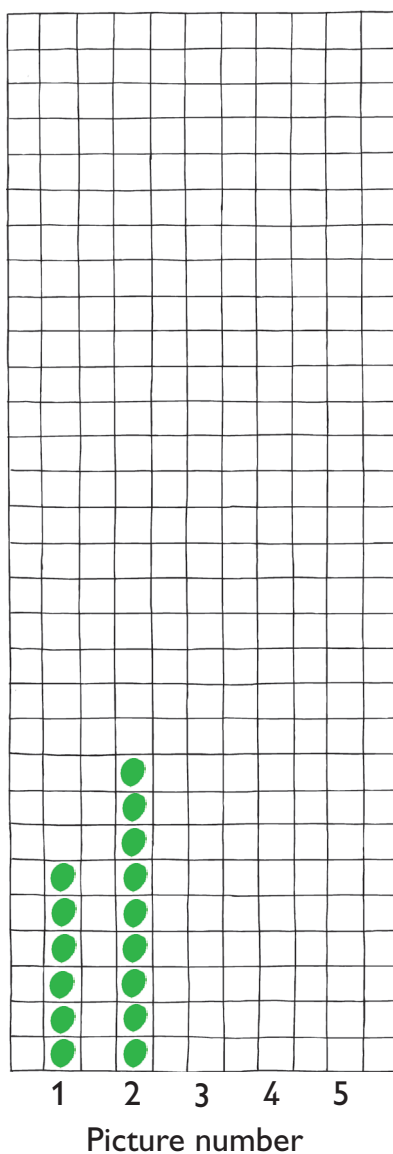1. Draw the fourth and fifth pictures.

2. Complete the table.

Picture number	1	2	3	4	5	6	7	10	12
Number of dots	6	9	12						
Number of crosses	2	6	12						

3. Complete each graph.

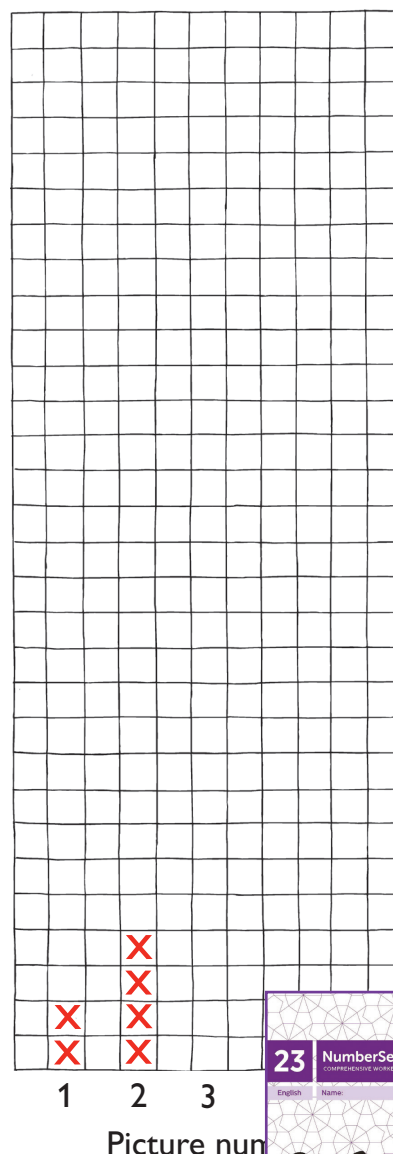*Carla is drawing a graph to show the number of dots in each picture. Danny is drawing a graph to represent the number of crosses in each picture.*

4. What is similar and what is different between the two graphs? Discuss.

1. Determine the sum of the following sets of consecutive numbers.

    Try to use an efficient strategy.

    a. 1 + 2 + 3 + 4

    b. 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8

    c. 15 + 16 + 17 + 18

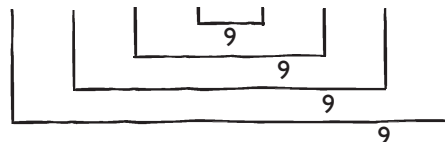   d. 15 + 16 + 17 + 18 + 19 + 20 + 21

Yusuf

*The first one was easy.
1 + 2 + 3 + 4 = 10.
I just added the numbers together.*

Ferial

I did not feel like adding all the numbers together, but I noticed that I could use a pairing strategy.

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8

Four groups of 9, and 4 times 9 equals 36.

Vusi

For the third one, I could also have just added them, however using Ferial's pairing strategy, there are just 2 lots of 33 which is 66.

15 + 16 + 17 + 18

Casey

I tried pairing but with an odd number of values it did not work so well, so I wrote the list of numbers to be like this:

15	16	17	18	19	20	21
21	20	19	18	17	16	15
36	36	36	36	36	36	36

*Which is 7 × 36 but I have each value twice so I must divide by 2, and
7 × 36 ÷ 2 = 126*

Dan

What I notice about Casey's approach is that it looks like she has added the first and last values, multiplied by the number of values and divided that by 2.
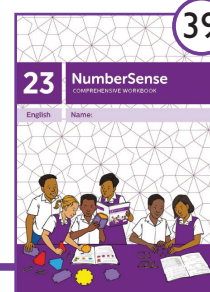
2. Use the strategy that makes the most sense to you to calculate.

    a. 1 + 2 + 3 + ... + 22 (all of the numbers from 1 to 22)

    b. 50 + 51 + 52 + 53 + 54 + 55 + 56 + 57 + 58 + 59 + 60

23 **NumberSense**
COMPREHENSIVE WORKBOOK
English   Name:

1. Lebo makes pictures with square tiles. The first three pictures make a pattern.

    Picture 1   Picture 4

    Picture 2   Picture 5

    Picture 3

    a. Draw the fourth and fifth pictures in the pattern.
    b. Complete the table and the flow diagram.

Picture no.	1	2	3	4	5
No. of tiles	3	4			

    Pic no. → ⬚ → ⬚ → No. of tiles
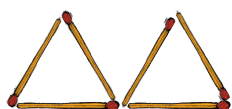
    c. What picture number can Lebo make with:
        *   15 tiles?   *   81 tiles?   *   27 tiles?
    d. Describe how you calculated the answers to c.
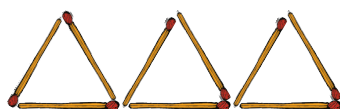
2. Suzi makes pictures with matches. The first three pictures make a pattern.

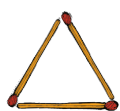   Picture 1   Picture 2   Picture 3   Picture 4   Picture 5

    a. Draw the fourth and fifth pictures in the pattern.
    b. Complete the table and the flow diagram.
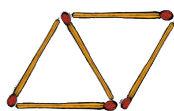
Picture no.	1	2	3	4	5
Matches	3	6	9		

    Picture no. → ⬚ → No. of matches

    c. What picture number can Suzi make with:
        *   15 matches?   *   81 matches?   *   27 matches?
    d. Describe how you calculated the answers to c.

4

24 **NumberSense**
COMPREHENSIVE WORKBOOK
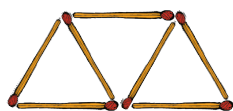
English   Name:

3. Viv makes pictures with matches. The first three pictures make a pattern.

Picture 1   Picture 2   Picture 3   Picture 4   Picture 5
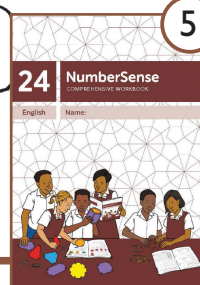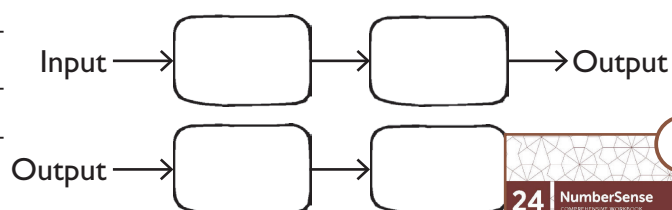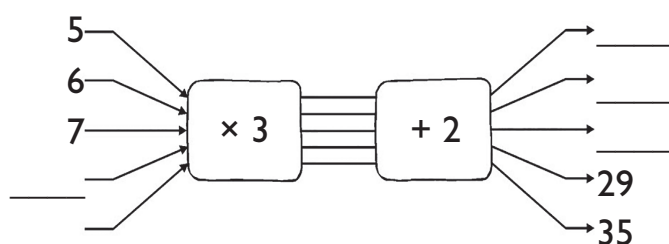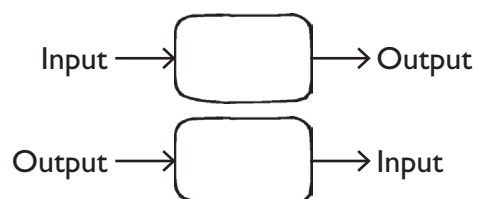
a. Draw the fourth and fifth pictures in the pattern.

b. Complete the table and the flow diagram.

Picture no.	1	2	3	4	5
Matches	3	5			

Pic no. → ☐ → ☐ → No. of matches

c. What picture number can Viv make with:

- 15 matches?
- 81 matches?
- 27 matches?

d. Describe how you calculated the answers to c.

4. Complete.

14
15
16
___
___
→ **+ 6** →
___
___
___
24
25

Input → ☐ → Output

Output → ☐ → Input

7
8
11
___
___
→ **× 4** →
___
___
___
48
60

Input → ☐ → Output

Output → ☐ → Input

5
6
7
___
___
→ **× 3** → **+ 2** →
___
___
___
29
35

Input → ☐ → ☐ → Output

Output → ☐ → ☐
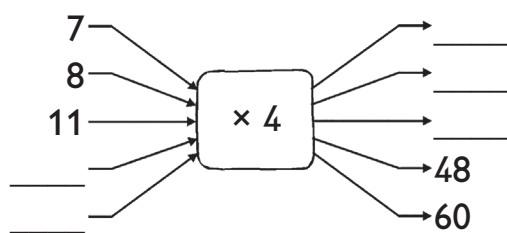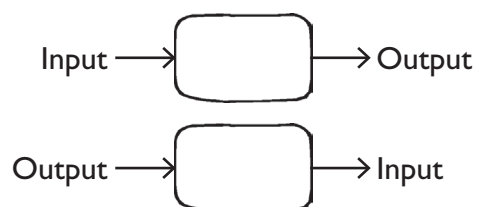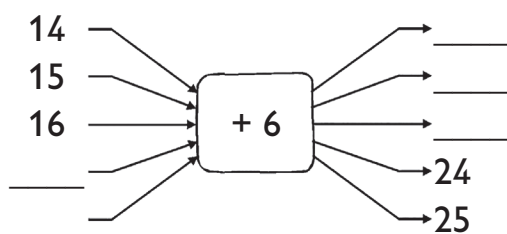
Task 1

Selecting the best savings option.

Palesa's parents have agreed to help her save. They have suggested three options and have asked her to select the option she wants.

Option 1   Her parents will open an account with an initial deposit of R120 and will deposit R5 into the account at the end of every month after that.

Option 2   Her parents will open an account with no initial deposit and will deposit R15 into the account at the end of each month.

Option 3   Her parents will open an account with no initial deposit. At the end of the first month they will deposit 1 c into the account. At the end of each month after that they will deposit $1\frac{1}{2}$ times the amount they deposited the previous month. So:

- At the end of month 2 they will deposit $1 \times 1\frac{1}{2} = 1,5$ c

- At the end of month 3 they will deposit $1,5$ c $\times 1\frac{1}{2} = 2,25$ c

And so on.

Help Palesa decide on the best option by completing the tasks.

1. Option 1

   a. Complete the table for the first 6 months.

Month	0	1	2	3	4	5	6	12	18	24	30
Balance (R)	120	125	130								

   b. Draw a flow diagram that you can use to calculate the balance in the account after any number of months. Use your flow diagram to determine the balance after 12, 18, 24 and 30 months and record your answers in the table.

2. **Option 2**

   a. Complete the table for the first 6 months.

Month		0	1	2	3	4	5	6	12	18	24	30
Balance (R)												

   b. Draw a flow diagram that you can use to calculate the balance in the account after any number of months. Use your flow diagram to determine the balance after 12, 18, 24 and 30 months and record your answers in the table.

3. **Option 3**

   a. Use a calculator to complete the table (round all values to three decimal places)

Month	Amount deposited	Balance (R)
0	0	0
1	0,01	0,01
2	0,015	0,025
3	0,023	0,048
4	0,034	0,081
5	0,051	0,132
6	0,076	0,208
7	0,114	0,322
8	0,171	0,493
9	0,256	0,749
10	0,384	1,133

Month	Amount deposited	Balance (R)
11	0,577	1,710
12	0,865	2,575
13	1,297	3,872
14	1,946	5,819
15	2,919	8,738
16	4,379	13,117
17	6,568	19,685
18	9,853	29,538
19	14,779	44,317
20	22,168	66,485
21	33,253	99,738

Month	Amount deposited	Balance (R)
22	49,879	149,617
23	74,818	224,435
24	112,227	336,662
25	168,341	505,003
26	252,512	757,515
27	378,768	1 325,666
28	568,151	2 177,893
29	852,227	3 456,234
30	1 278,340	5 373,744
31	1 917,511	8 250,010
32	2 876,266	12 564,409

b. Use your table to answer the questions.

- How much did Palesa's parents deposit in 5th month?

- In which month did the balance exceed R1?

- In which month did the balance exceed R20?

4. Use the values that you have calculated to complete the graph.
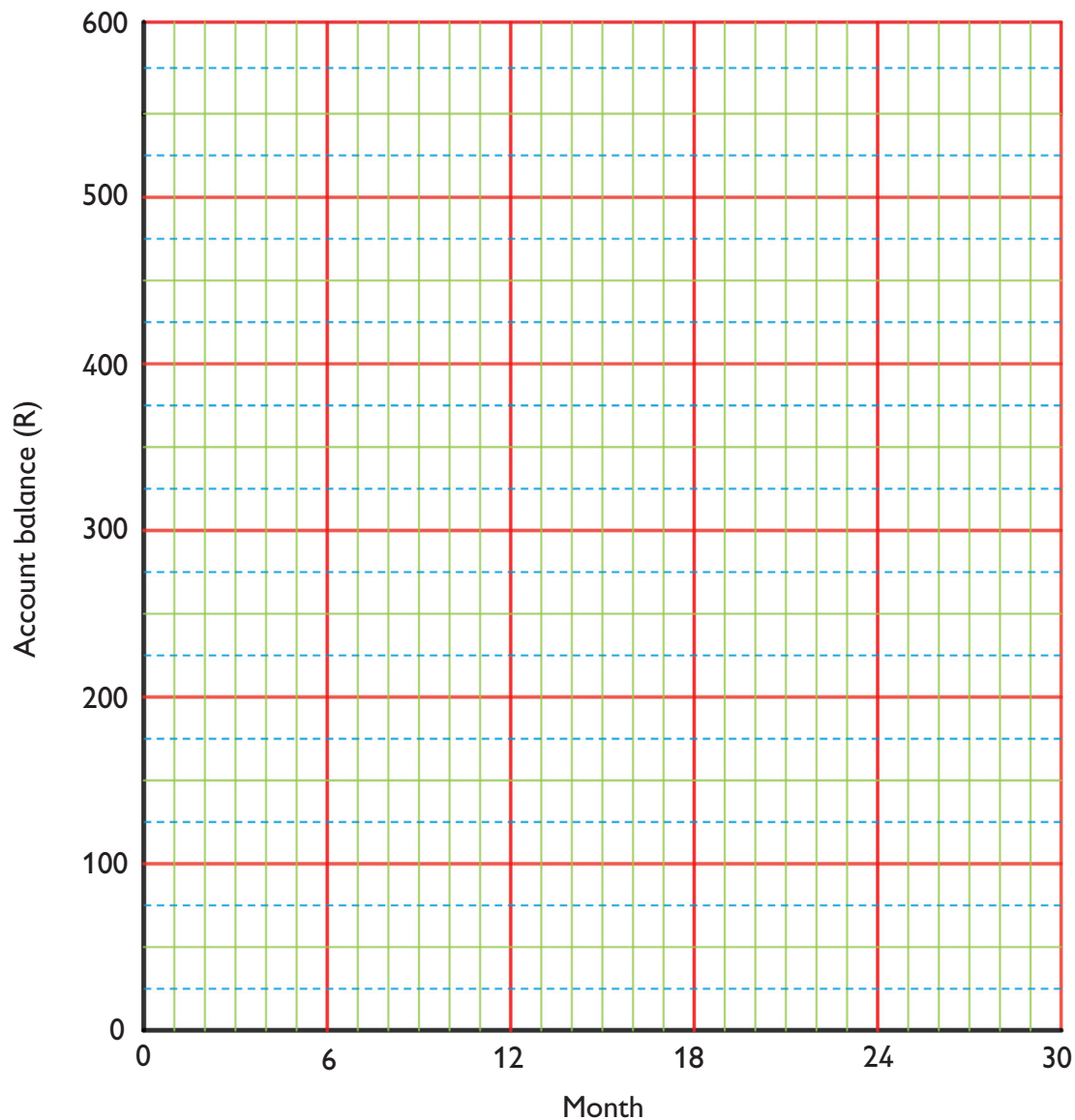
5. Use your graph to determine when:

- option 1 is the best option.

- option 2 is the best option.

- option 3 is the best option.

Exploration with cubes and squares

Formulae that we have already developed over time.

- direct proportion

1	2	3	4	n
3	6	9	12	$3n$

+ 3   + 3   + 3

1	2	3	4	n
d	$2d$	$3d$	$4d$	dn

+ d   + d   + d

- linear

1	2	3	4	n
5	8	11	14	$3n + 2$

+ 3   + 3   + 3

1	2	3	4	n
$a + d$	$a + 2d$	$a + 3d$	$a + 4d$	$dn + a$

+ d   + d   + d

- sum of consecutive whole numbers

$$1 \qquad\qquad = 1$$
$$1 + 2 \qquad = 3$$
$$1 + 2 + 3 \quad = 6$$
$$1 + 2 + 3 + 4 = 10$$

$$1 + 2 + 3 + \ldots + n = \frac{n \times (n + 1)}{2}$$

See Workbook 14 p.28
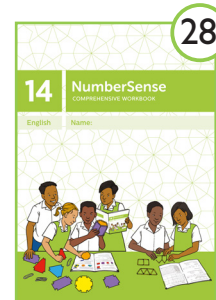
- sum of consecutive odd numbers

$$1 \qquad\qquad = 1$$
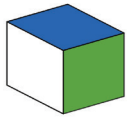$$1 + 3 \qquad = 4$$
$$1 + 3 + 5 \quad = 9$$
$$1 + 3 + 5 + 7 = 16$$

$$1 + 3 + 5 + \ldots + 2n - 1 = n^2$$
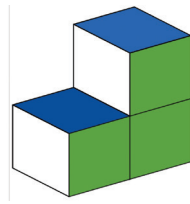
See Workbook 25 p.24
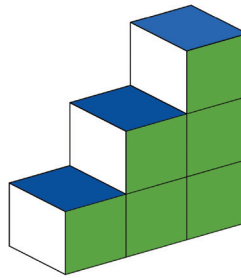
1. Thato is building a series of shapes with cubes. The shapes in the series make a pattern.
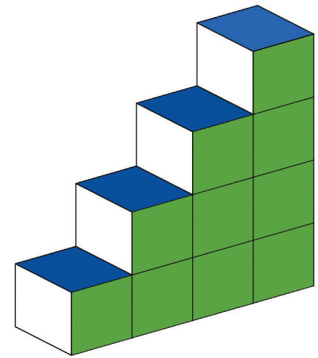

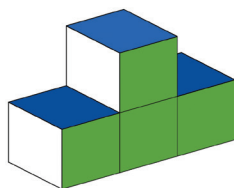shape 1   shape 2   shape 3   shape 4

a. Use the squares from the GeoGenius Construction Kit to make the first four shapes in Thato's series of shapes.

b. Use the shapes you have built to develop a formula for the number of cubes in the nth shape of the series.

c. Use the shapes that you have built to develop a formula for the number of squares in the nth shape in the series. That is, the total surface area of the shape.
   Hint: First develop formulae for the green squares, the blue squares and the white squares.
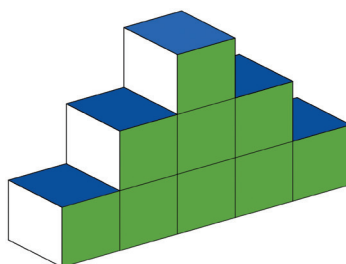
2. Sindi is building a series of shapes with cubes. The shapes in the series make a pattern.
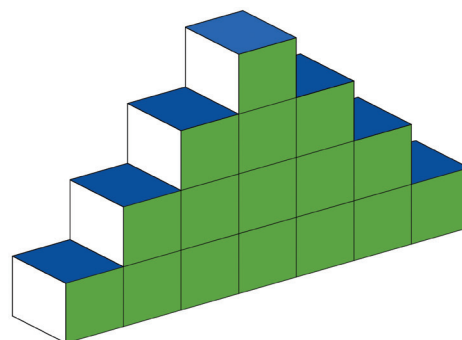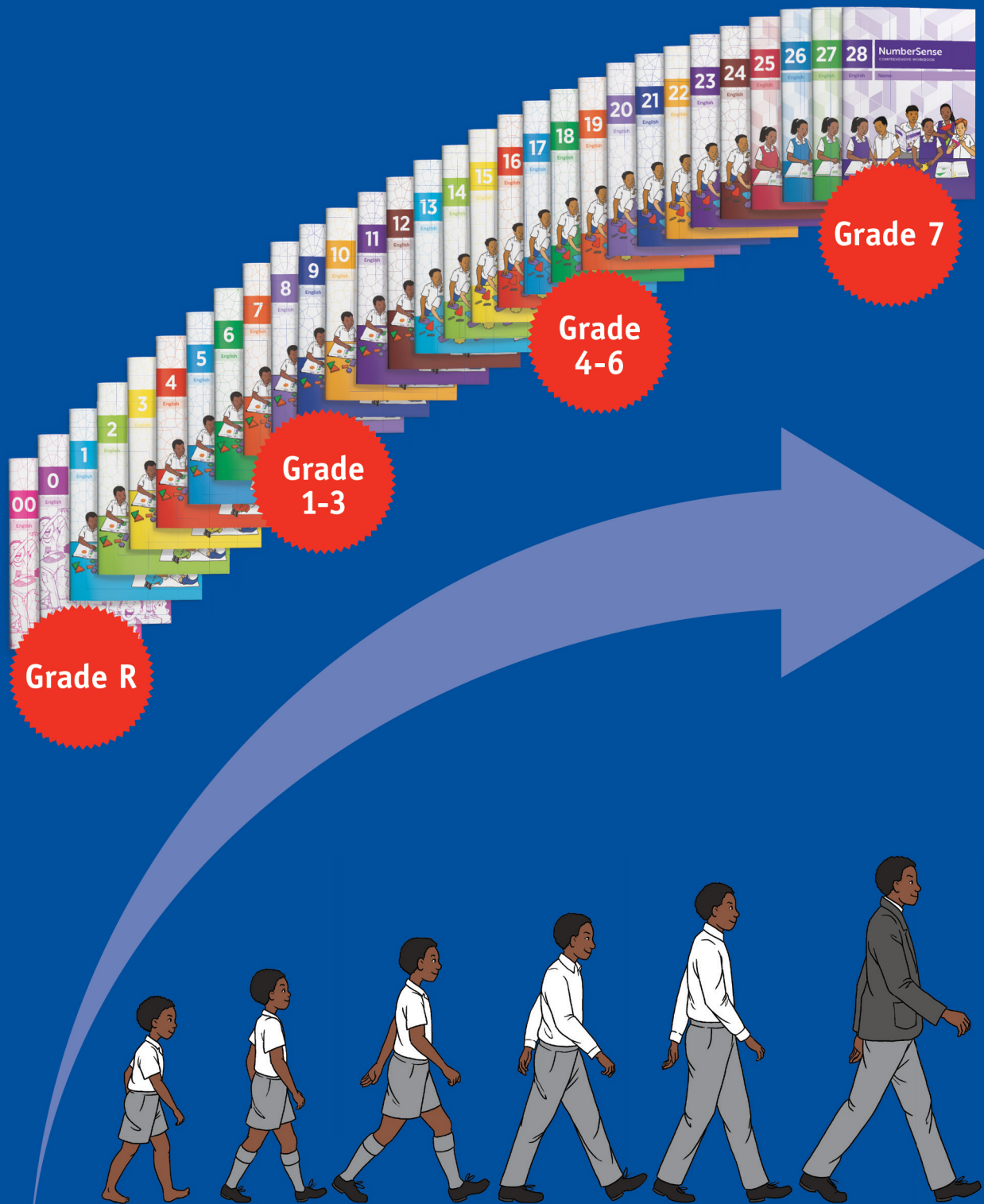

shape 1   shape 2   shape 3   shape 4

    a. Use the squares from the GeoGenius Construction Kit to make the first four shapes in Sindi's series of shapes.

    b. Use the shapes you have built to develop a formula for the number of cubes in the nth shape of the series.

    c. Use the shapes that you have built to develop a formula for the number of squares in the nth shape in the series. That is, the total surface area of the shape.
       Hint: First develop formulae for the green squares, the blue squares and the white squares.

NUMBERSENSE COMPREHENSIVE WORKBOOK JOURNEY

Grade 7

Grade 4-6

Grade 1-3

Grade R

Brombacher & Associates