MEASUREMENT

FIRST DRAFT

TEACHER GUIDE

CHAPTER 6

Brombacher & Associates

MEASUREMENT

Measurement has its origin in the desire to compare, who is taller and who is shorter (length); which container holds more and which holds less (capacity/volume); which object is heavier and which is lighter (mass/weight); which object takes up more space and which takes up less space (area and volume) etc.

Measurement is one of the most practical topics in the mathematics curriculum. The activities in the NumberSense Mathematics Programme assume a very practical, hands-on approach.

Measurement involves assigning a numerical value to an attribute to enable comparison, ordering, and calculation. The attributes that the NumberSense workbooks focus on are: length, capacity (volume), mass (weight), area, and time.

There are three key aspects to measuring:

- knowing what attribute is being measured (e.g. what is mass (weight)?).
- being able to describe the attribute that is being measured (e.g. how do you describe how heavy something is?).
- the ability to use the appropriate measuring instrument(s) to measure the attribute (e.g. how do you use a scale?).

A developmental trajectory for developing measuring skills

There are typically four key developmental stages in learning to measure. It is important that learners experience each of these stages rather than simply learning how to use a measuring instrument and performing calculations.

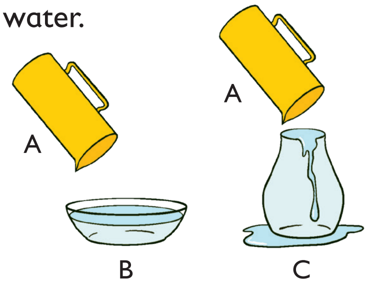
These stages are:

Direct comparison

Direct comparison involves considering two or more objects and comparing them in terms of the attribute of interest. By putting two objects next to each other, we can determine which is longer and which is shorter (length). By transferring the contents of one container to another, we can determine which holds more and which holds less (volume/capacity).

Who is taller, Palesa or Lesego?

Arrange the containers A, B, and C, from the one that holds the least water to the one that holds the most water.



Indirect comparison

Because it is not always possible to compare two objects directly (e.g. by holding them next to each other), we can use a third object that can be compared directly to each of the two objects to act as a reference. If you can cover one tile with your hand and not a second tile, then the second tile is larger (has a greater surface area) than the first. If a stick is shorter than one object and longer than a second object, it follows that the first object is longer than the second. Direct and indirect comparison does not, however, tell us by how much one object is more or less than the other.

Measuring with non-standard units

Mary determined a table to be 12 handspans long. Sipho determined the same table to be 14 handspans long. Discuss why the values that they determined were different.

One of the most important reasons for learners to be involved in activities that involve indirect comparison is to develop an awareness for the need of a unit with which to measure. To assign a number to an attribute of an object allows us to begin to answer questions such as "Which is heavier?" and "By how much?"

Learning to measure using non-standard units is important because in doing so learners are learning what is means to measure. They are learning to select a unit and to determine how many of those units are needed to describe the attribute of the object being measured.

Measuring with non-standard units draws attention to the need to select with care the unit used to measure an attribute. It should be selected in relation to the "amount" of the attribute being measured. It makes no sense to measure the volume (capacity) of a bucket using teaspoons or cups and it makes no sense to measure the length of the school field using match sticks.

Measuring with non-standard units also develops the awareness for the need to be able to convert between units.

The inefficiencies associated with using non-standard units paves the way for the introduction of more standardised units and measuring instruments.

For example, if two learners measure the width of the same doorway using their pencils, they may get different results because their pencils are not the same. Experiences such as these make learners aware that they cannot compare their findings unless the units they use are the same.

Measuring with standard units

It is because of an increasing awareness of the limitations of non-standard units that standard units have been introduced into measurement over time.

Standard units have two important characteristics. First, they are well-defined (standardised). Secondly, standard units can typically be divided into smaller standard units and combined into larger standard units. For example, 1 kg = 1 000 g and 1 000 kg = 1 tonne; 10 mm = 1 cm, 100 cm = 1 m and 1 000 m = 1 km etc.

The ability to divide standard units into smaller standard units and combine them into larger standard units is important because it allows for the use of units that are appropriate to (efficient) the object being measured.

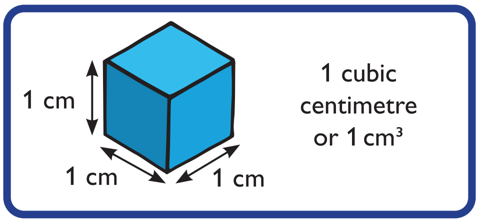
Standard units introduce measuring instruments associated with them. Rulers and measuring tapes to measure and describe length in terms of centimetres and metres; scales to measure and describe mass/weight in terms of grams and kilograms; measuring jugs to describe capacity/volume in terms of millilitres and litres, and square grids to describe the area of shapes in terms of square centimetres etc.

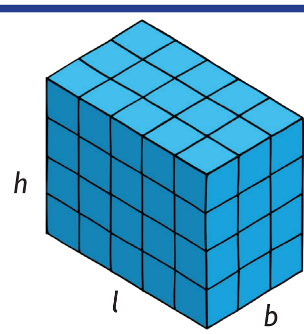
The *volume* of a 3D object is the amount of space taken up by an object. Volume is reported in cubic millimetres (mm^3) - a cube with 1 mm edges, cubic centimetres (cm^3) - a cube with 1 cm edges, cubic metres (m^3) - a cube with 1 m edges, and so on.

The formula for the volume of a rectangular prism is:

volume = number of cubes in a of layer × number layers

or $V = l \times b \times h$





Measuring instruments also introduce the need to develop the skill to use these instruments and the ability to convert between different units.

Finally, learners will begin to use formula to calculate attributes. For example, formulae to calculate the areas of rectangles, squares, and circles etc. in terms of attributes such as length, height and radius etc.

The development of measuring skills in the NumberSense Mathematics Programme

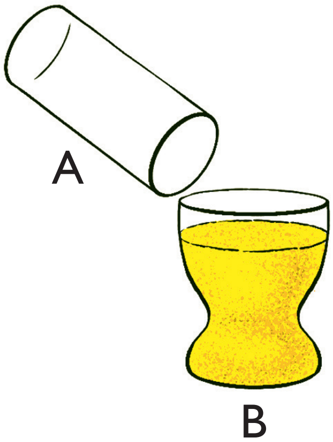
The nature of the measuring activities in the NumberSense Mathematics Programme is informed by the developmental trajectory described above. In broad terms:

- Measurement in Grade R and 1 (Workbooks 00 to 4) focuses on using direct comparison to compare and order.
- Measurement in Grades 2 and 3 (Workbooks 4 to 9) introduces indirect comparison to compare and order.
- In Grade 3 (Workbooks 9 to 12), non-standard units are introduced. The nature of the activities draws attention to the need for selecting appropriate units and the challenges associated with non-standard units.
- From Grade 4 (Workbooks 13 and up), the workbooks begin the transition to standard units and converting between these. By Grade 6 and 7, the workbooks begin to introduce formulae associated with perimeter, area, volume and so on.

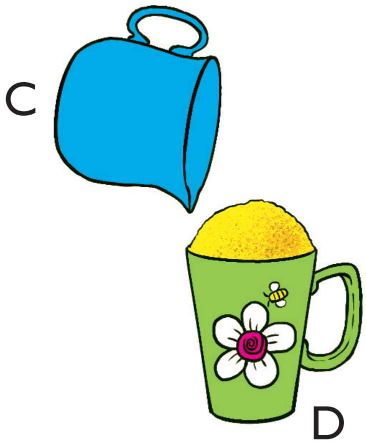
Throughout the NumberSense Mathematics Programme, measurement activities are used as a context for exploration, sense-making and problem solving.

For each pair of containers your teacher gives you, use sand to help you decide which container holds more.

1. Which container holds more sand, A or B?

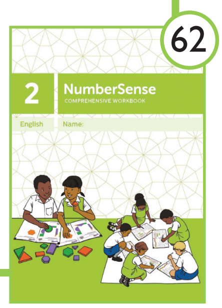


2. Which container holds more sand, C or D?

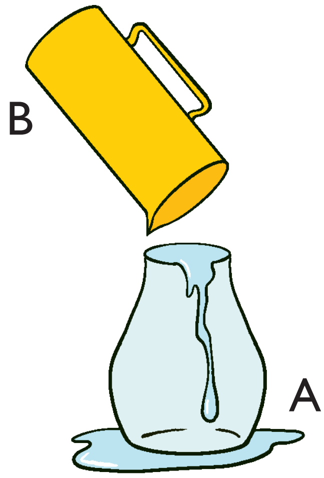


3. Which container holds more sand, E or F?

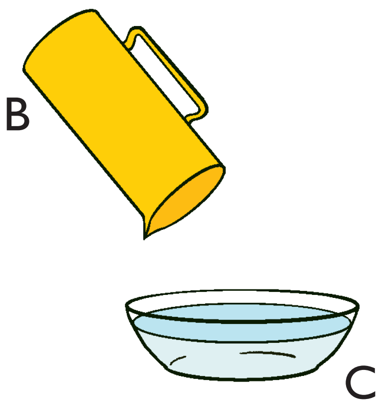
Measurement



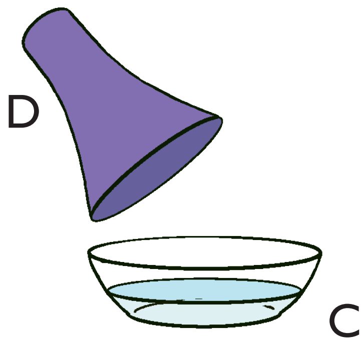
Use water to help you arrange the bottles your teacher gives you in order from the one that holds the least water to the one that holds the most water.



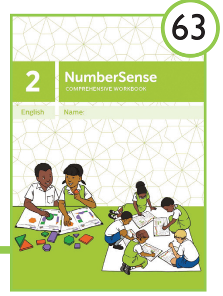
1. Which container holds more water, A or B?



2. Which container holds more water, B or C?



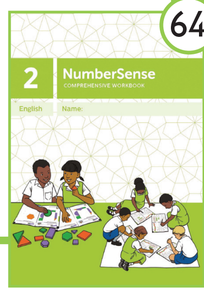
3. Which container holds more water, C or D?



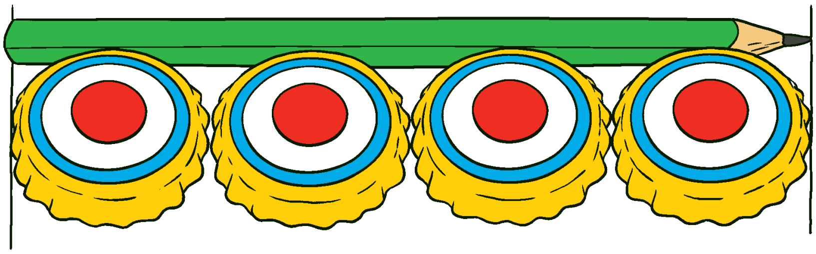
Emma wants to determine which holds more water: the basin in her bathroom or the sink in the kitchen. This is what she did.

1. Which holds more water: the bathroom basin or the kitchen sink? Why do you say so?

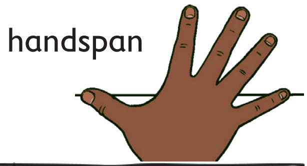
2. Which container: the cup, the larger bucket or the smaller bucket was most helpful? Explain why.



Length is the term used to describe the size of an object from one point to another point.



We can say the length of the pencil is 4 bottle tops.

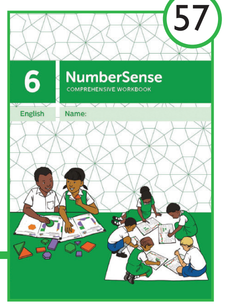


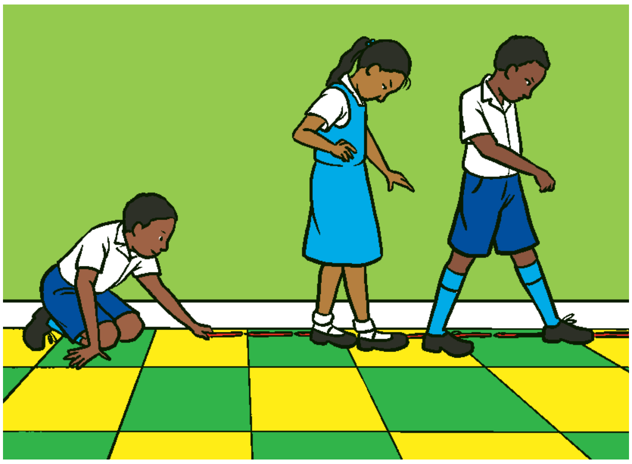
1. Complete.

	Number of matchsticks	Number of bottle tops	Number of handspans
Length of a book	more than 6	10	nearly 2
Length of a table	33	50	nearly 9

2. What did you notice about using the matchsticks, bottle tops and handpans to determine:
   - The length of the book?
   - The length of the table?

3. Mary determined a table to be 12 handspans long. Sipho determined the same table to be 14 handspans long. Discuss why the values that they determined were different.





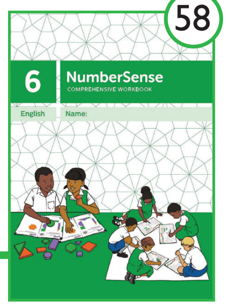
Compare the lengths of two different rooms in your school using pencils, footsteps and strides.

1. Complete.

	Number of pencils	Number of footsteps	Number of strides
Length of room one	32	25	more than 4
Length of room two	42	33	less than 6

2. What did you notice about using these objects to determine the length of the classrooms?
   - Pencils?
   - Footsteps?
   - Strides?

3. Fundi determined the length of a room to be 8 strides. Jan determined the length of the same room to be 11 strides. Why were the values that they determined different?



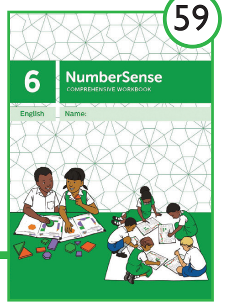
1. Determine the length of your school bag, your desk and your teacher's board using bottle tops, matchsticks and handspans and complete the table.

	Number of bottle tops	Number of matchsticks	Number of handspans
Length of bag	15	10	nearly 3
Length of desk	23	15	4
Length of board	120	75	20

2. Which object was most useful and least useful for determining the length of:
   - Your bag? Explain.
   - The desk? Explain.
   - Your teacher's board? Explain.

3. Thembi measured the length of a table using matchsticks. She says that the table is 32 matchsticks long. Mary measured the length of the same table using pencils. She says that the table is 8 pencils long.

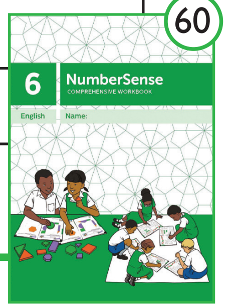
   Mary measured the length of a school bag to be 2 pencils long. What is the length of this bag in matchsticks?



The children in Zoliswa's class are comparing their handspans by counting how many pencils they can cover with their handspan.

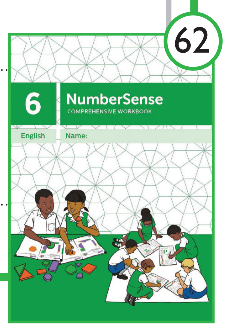
1. Use the table below to record the number of pencils the children in your class can cover with their handspans.

Name	Pencils	Name	Pencils	Name	Pencils
Aqhakama	19	Aiden	18	Boitumelo	22
Qhama	19	Marriam	21	Onele	19
Aqeefa	19	Phoebe	20	Sihle	22
Gouwla	22	Jensen	23	DeAngelo	21
Isabella	20	Hlalumi	21	Thatenda	22
Zackary	23	Khazimla	26	Areez	20
Hopeson	21	Ibraheem	20	Emmanuel	24
Misokhuhle	21	Ionde	21	Othalviwe	23
Ignacia	23	Onke	17	Raaziah	[illegible]
Ghaliema	24	Sibulele	19	Jude	[illegible]



First, I look for the person whose handspan covers the smallest number of pencils: that is Jamie with 14 pencils. Next, I look for the person whose handspan covers the largest numer of pencils: that is Sinalo with 24 pencils. Then I draw a grid and I divide it up into 11 spaces and I then stick a picture for each child.

2. Repeat what Zoliswa has done using your own information. Use the pictures provided and the space below.



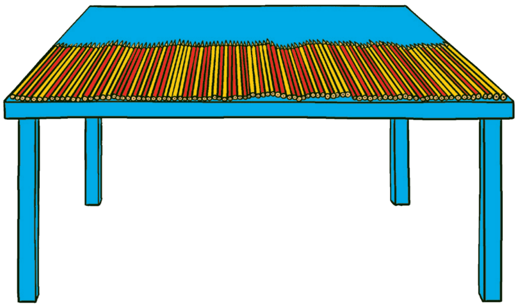
Study the summary that you have made on page 62 using the information for your class.

1. What is the smallest number of pencils covered by a handspan?

2. What is the largest number of pencils covered by a handspan?

3. Are there any numbers of pencils between the smallest and largest number not covered by the handspans of the children in your class?

   Zoliswa measured the width of a table using pencils. She needed 95 pencils.

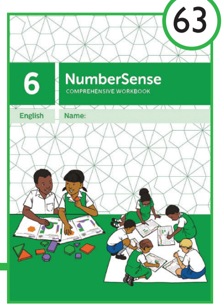


4. Approximately how many handspans wide would most of the children in your class say the table is? Explain your answer.

5. What is the largest number of handspans that somebody in your class would need to measure the width of the table?

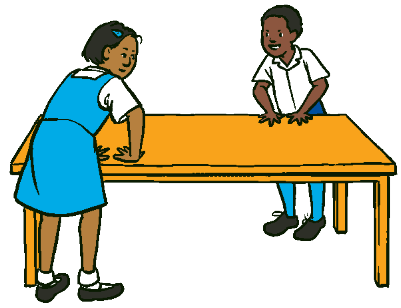
6. What is the smallest number of handspans that somebody in your class would need to measure the width of the table?

Compare your solutions with a friend.



In Zoliswa's class:

- Nozuko's handspan covers 10 pencils
- Wandile's handspan covers 18 pencils
- Clare's handspan covers 20 pencils
- Salim's handspan covers 25 pencils



1. Nozuko measures a table and says that it is 5 handspans wide. Wandile measures the same table using handspans. Approximately how many handspans wide does Wandile say the table is?

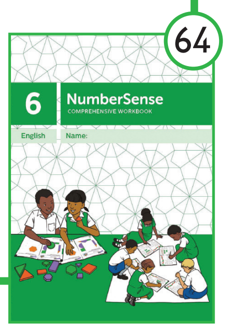
2. How many handspans wide would Clare say the table is?

3. How many handspans wide would Salim say the table is?

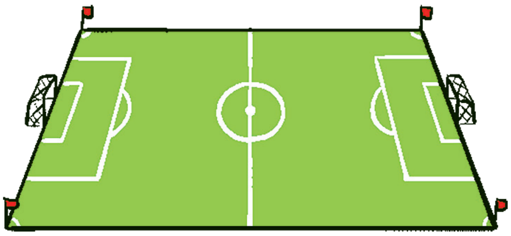
4. What are the advantages and disadvantages of measuring the width of a table using pencils?

5. What are the advantages and disadvantages of measuring the width of a table using handspans?

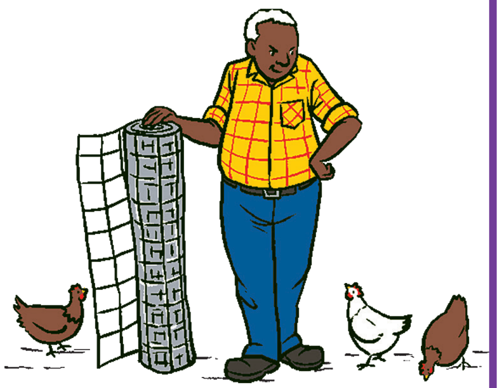
Discuss with your friends.

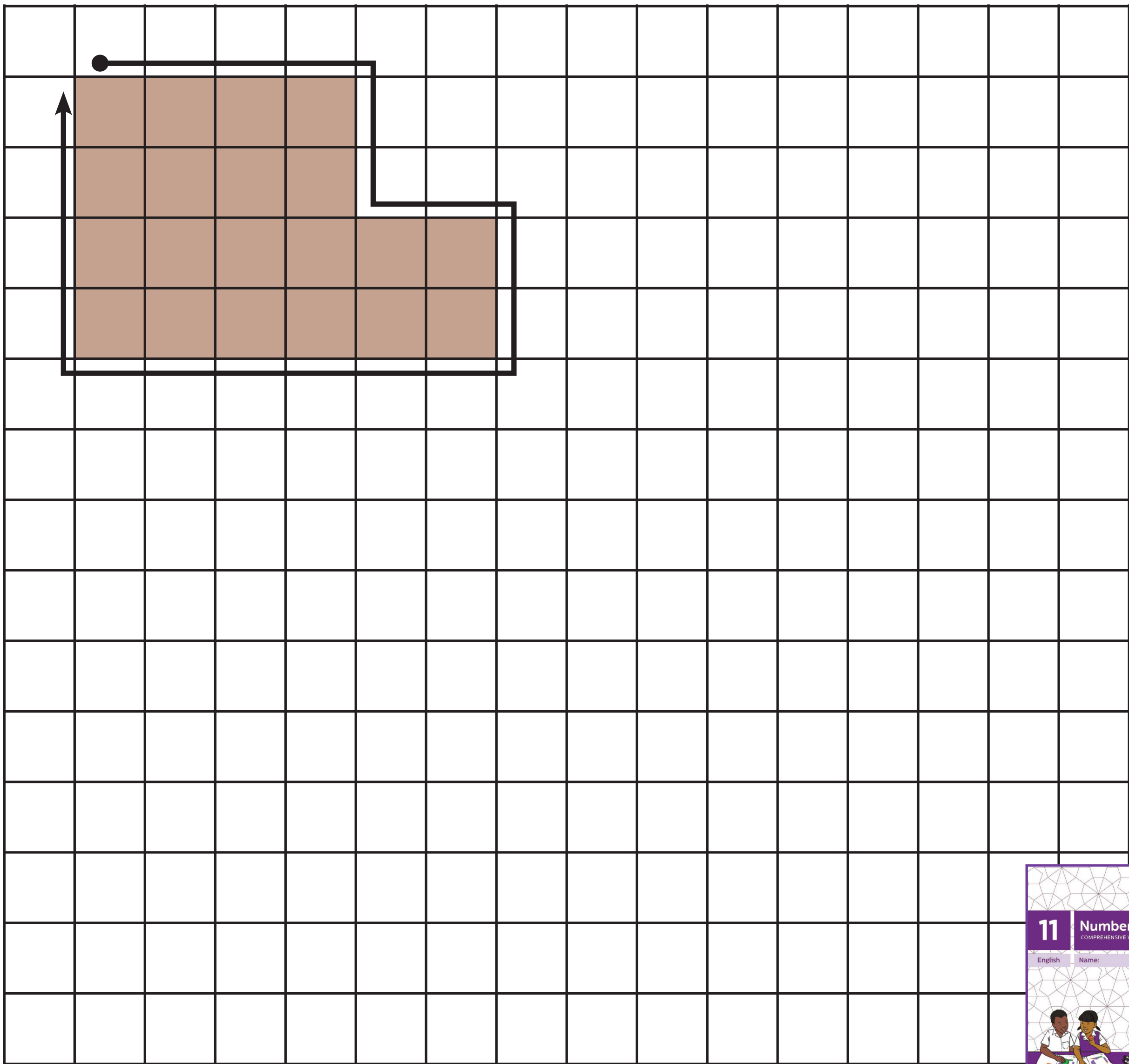


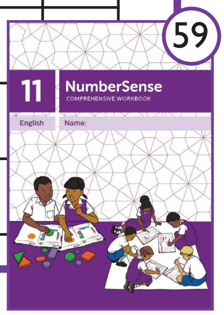
1. A soccer field is 100 metres long and 50 metres wide. How many metres does a player run if they run around the field:
   - Once?
   - 10 times?



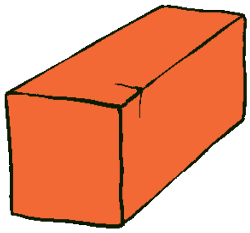
2. Mr Dlamini has a 20 metre roll of fencing that he can use to put around his chicken enclosure. He could use the fencing to build an enclosure as shown here.
   Use the rest of the grid to investigate other shapes that Mr Dlamini could build if he uses all 20 metres of his fencing.

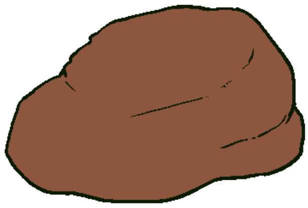






Thato and Matthew wanted to know if Thato's brick or Matthew's rock is larger. This is how they did it.





They filled a bucket with water.

Thato carefully dropped his brick in the water. Some water flowed out of the bucket.

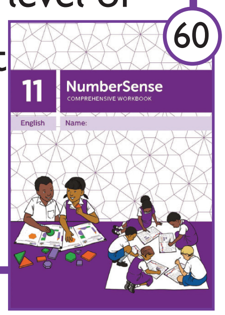
Thato then took his brick out the water and used a marker to mark the new level of the water on the side of the bucket.

Thato refilled the bucket with water.

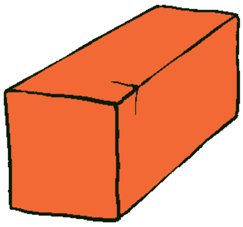
Matthew carefully dropped his rock in the water. Some water flowed out of the bucket.

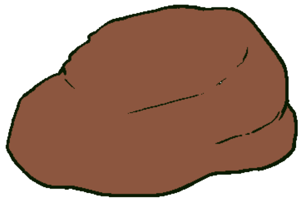
Matthew then took his rock out of the water and used a marker to mark the new level of the water on t

of the bucket.



1. • Which is larger, Thato's brick or Matthew's rock?

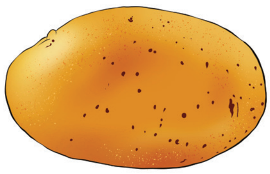




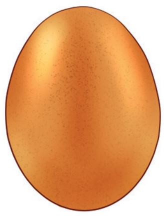
   • How do you know?

2. Which is larger, a milk bottle or a tin of beans?
   • Use Thatho and Matthew's method with a bucket of water. Which is larger?

   • Use Thatho and Matthew's method with a jug of water. Which is larger?
   • What do you notice about using the bucket and the jug? Which is better? Explain.

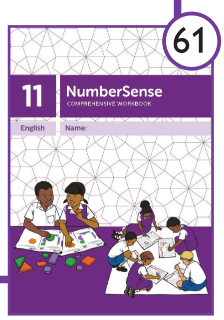


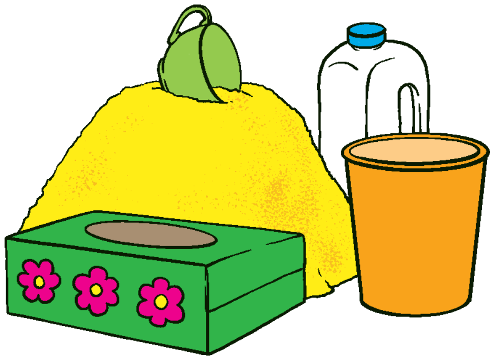
3. Which is larger, an egg or a small potato?
   • Use Thato and Matthew's method with a jug of water. Which is larger?



   • Use Thato and Matthew's method with a cup of water. Which is larger?
   • What do you notice about using the jug and the cup? Which is better? Explain.

Discuss with a friend.





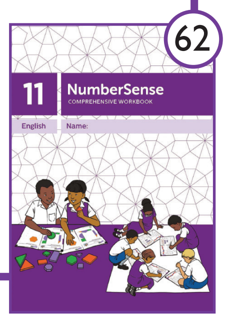
For the three containers that your teacher will provide, compare the capacity by counting the number of cups of sand that fill the containers.

The table has the results from three groups who compared the capacity of a yoghurt tub, a tissue box and milk bottle using cups of sand.

	Group 1	Group 2	Group 3
Yoghurt tub	2 cups	1 cup	4 cups
Tissue box	5 cups	More than 2, less than 3 cups	9 cups
Milk bottle	7 cups	More than 3, less than 4 cups	14 cups

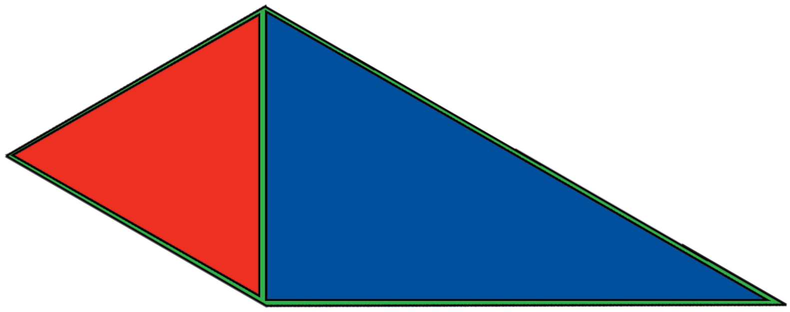
1. Compare the groups' results.
   - Which group used the largest cup? Explain why you say so.
   - Which group used the smallest cup? Explain why you say so.

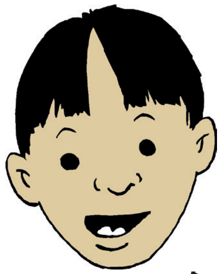
2. Caitlyn used 150 marbles to fill a jug and 300 marbles to fill a bowl. Caitlyn used 100 bottle tops to fill the jug. How many bottle tops will she need to fill the bowl? Explain.



MOSAIC PUZZLE

Abdul is playing with his Mosaic Puzzle pieces.





Use Abdul's thinking to complete the following. In each case, trace the pieces to justify the claim.

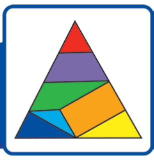
1. Piece 5 covers the same amount of space as piece 1 and piece _____.

2. Piece 3 covers the same amount of space as piece 5 and piece _____.

3. Piece 4 covers the same amount of space as piece _____ and piece _____.

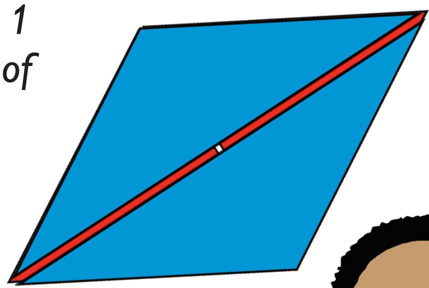
4. Piece 4 covers the same amount of space as 3 × piece _____.

MOSAIC PUZZLE



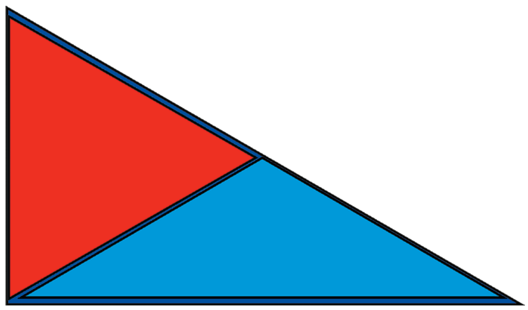
Sally and Xolile are trying to determine how many of piece 2 covers the same amount of space as piece 5.

I noticed that 2 of piece 1 cover the same amount of space as 2 of piece 2.



That means that piece 1 and piece 2 cover the same amount of space.

When I join piece 1 and piece 2, they fit exactly onto piece 5.

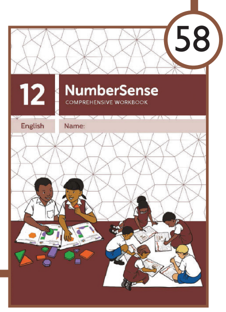


Because piece 1 and piece 2 cover the same amount of space, and because piece 5 is covered by pieces 1 and 2, we can conclude that two of piece 2 cover the same amount of space as piece 5. This is true even though we cannot actually cover piece 5 with two piece 2s.

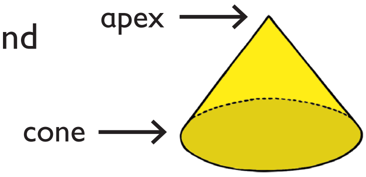
Use the same thinking to determine the following.

1. How many of piece 1 cover the same amount of space as piece 4? Trace the pieces to explain your answer.

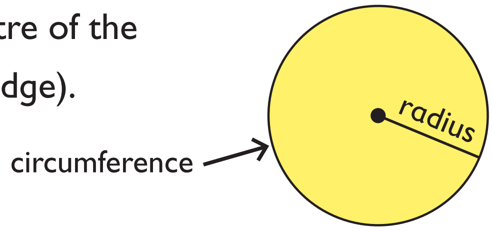
2. How many of piece 2 cover piece 3? Trace the pieces to explain your answer.



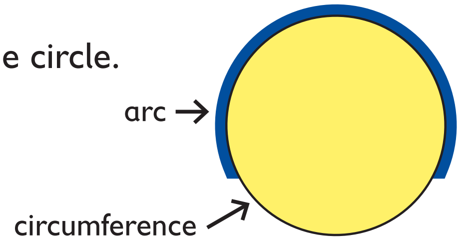
A three-dimensional shape made up of a circular base and a surface that makes a point (vertex), called the apex, is called a cone.



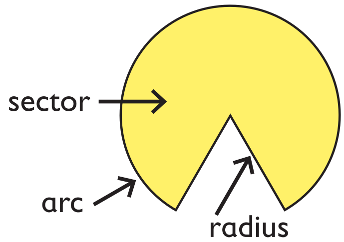
The radius of a circle is the distance from the centre of the circle to any point on the circle's circumference (edge).



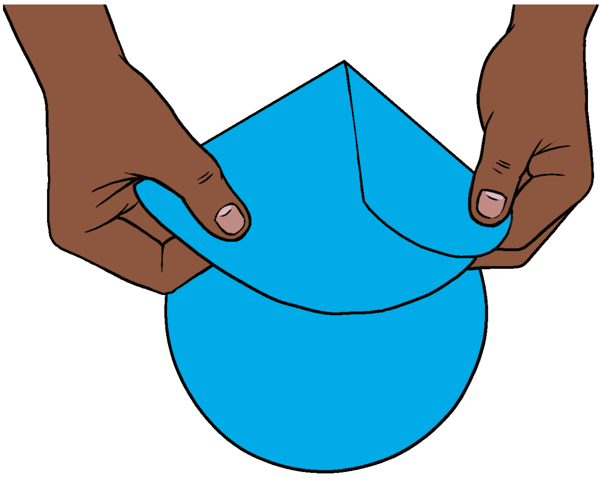
An arc of a circle is a part of the circumference of the circle.

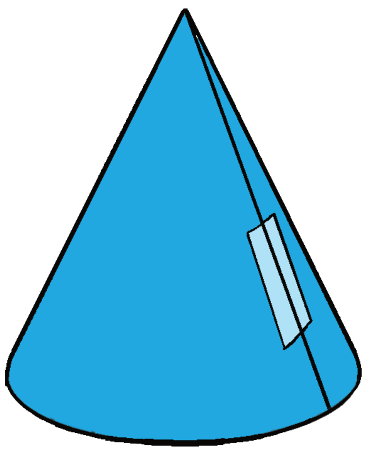


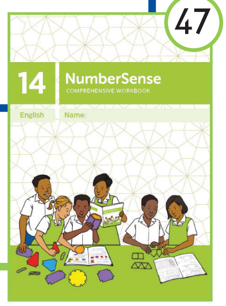
A sector of a circle is the part of the circle enclosed by two radii and an arc of the circle between the radii.



A cone can be constructed using a circular base and a sector of a circle.

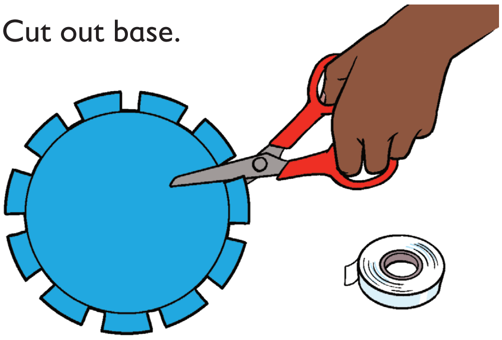




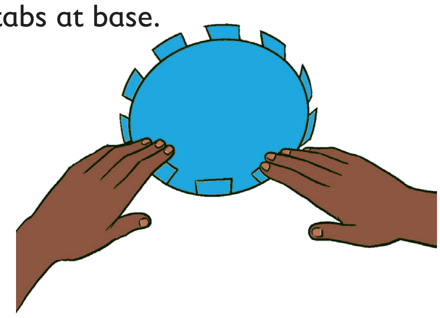


How to make a cone.

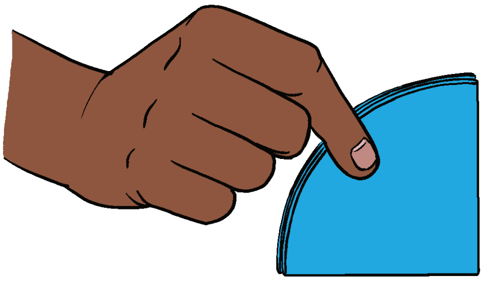
1. Cut out base.



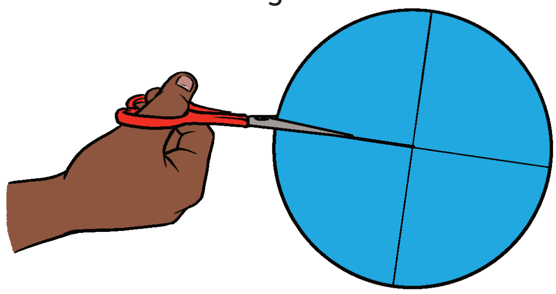
2. Fold tabs at base.



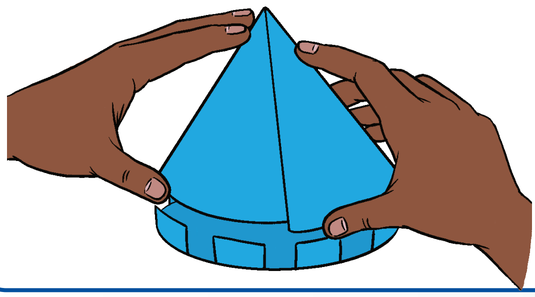
3. Fold circle to determine centre.



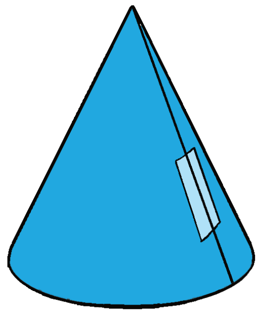
4. Draw and cut along radius.



5. Fold the circle to match the base.



6. Completed cone.



1. Note to teacher and parents: draw the circles and base on A4 paper, A3 paper or newsprint.

For this activity your teacher will give you:

- A template of a circle (radius 4 cm) with tabs to be used as the base of a cone.
- A template for 5 circles (radius 6 cm; 8 cm; 10 cm; 12 cm and 14 cm)

Use the step by step instructions for making cones.

1. Trace and cut out 5 bases with tabs and fold up the tabs.
2. Cut out each circle, fold it to determine the centre of the circle. Draw one radius from the centre of the circle to the perimeter of the circle and cut along the radius.
3. Use the 5 bases and 5 circles to make 5 different cones.

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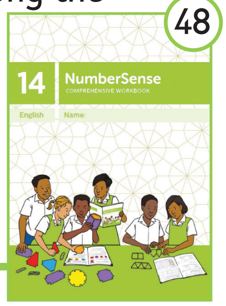
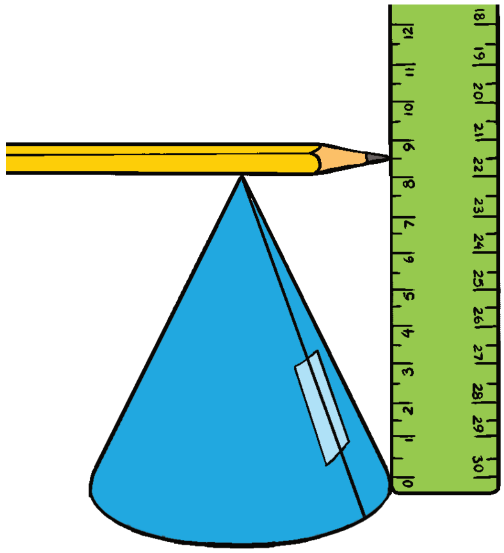


4. Measure the height of the cones that you made and complete the table.



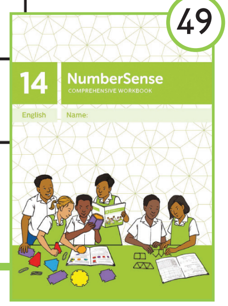
Radius of circle used for curved surface (cm)	6	8	10	12	14
Height of cone	4,5 cm	7 cm	9 cm	11 cm	13 cm

5. What do you notice about the relationship between the radius of the circle used for the curved surface of the cone and the height of the cone?

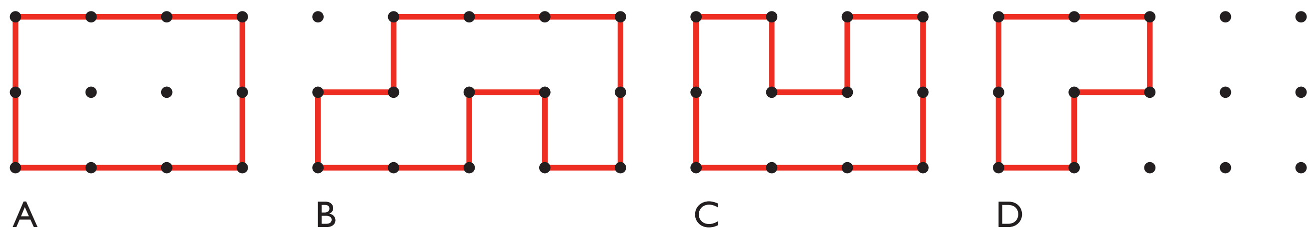
6. Study the table and predict the height of the cone built using a circular base with a radius of 4 cm and a circle with a radius of 16 cm. Build the cone and check your prediction.

7. Work with a group of friends and investigate what happens to the relationship between the radius of the base, the radius of the circle used for the curved surface and the height of the cone. Complete the table. What do you notice?

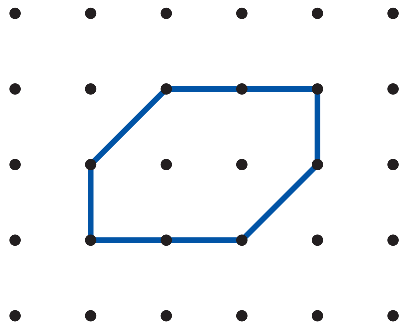
Name	radius of base	radius of curved surface			
		12	14	16	18
	6				
	8				
	10				



1. Order the shapes from the shape with the shortest perimeter to the shape with the longest perimeter. ______ ; ______ ; ______ ; ______



2. Diana calculated the perimeter of this shape like this.



2 + 1 + 1 + 2 + 1 + 1 = 8 units

Explain why Diana is not correct.

Without using a ruler, complete for the shape:

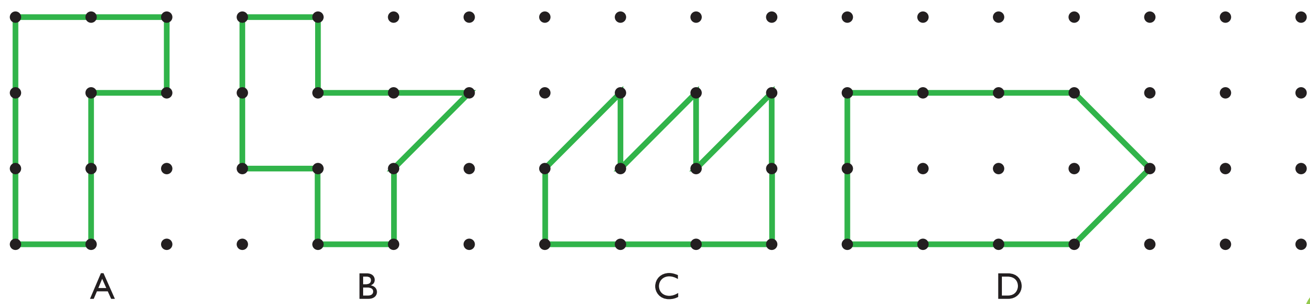
______ cm < perimeter of shape < ______ cm

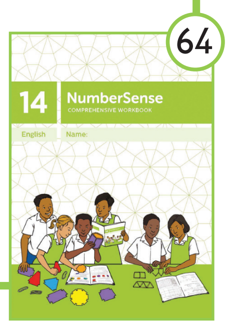
Explain how you did this.

3. Without using a ruler, arrange the shapes from the shape with the shortest perimeter to the shape with the longest perimeter.

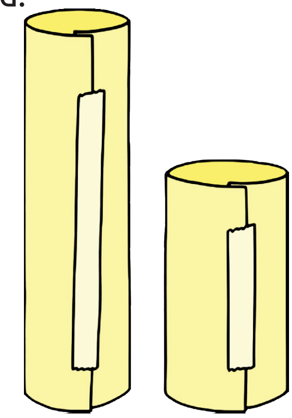
______ ; ______ ; ______ ; ______

Explain how you did this.





1. Make two cylinders as follows:
   - Fold and cut an A4 piece of card into two A5 pieces of card.
   - Use one of the pieces of card to make a cylinder by taping the long edges together.
   - Use the other piece of card to make a cylinder by taping the short sides together. Make sure that the overlap at the join is the same for both cylinders.



2. Which cylinder do you think will hold more when placed upright: the long one or the short one?

The measure of how much space is contained within, or occupied by a 3-dimensional object is called the volume.

3. Use beans to determine which cylinder has the greater volume.
   a. Explain how you did it.

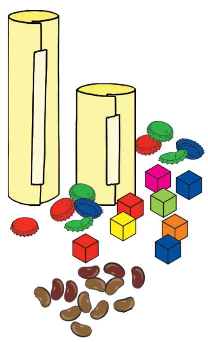
   b. Does your approach tell you how much more this one holds than the other or only which one holds more? Discuss.

4. Now, determine how many beans, bottle tops and cubes fill each cylinder. Complete the table.

	Number of beans	Number of bottle tops	Number of cubes
Long cylinder			
Short cylinder			

5. Based on the values in the table, determine how much greater the volume of the larger cylinder is compared to the other. Express your finding using fractions.

6. Which objects: beans, bottle tops or cubes, worked best to determine the volume of the cylinders? Explain.



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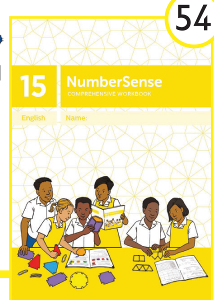
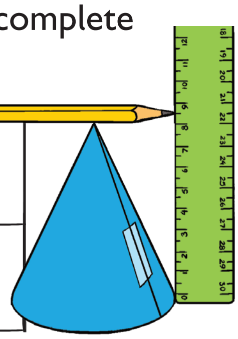


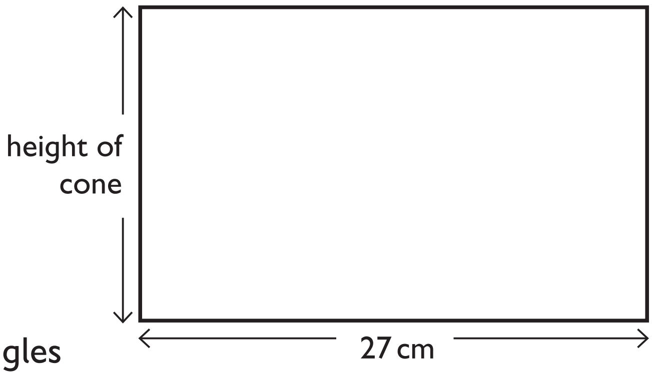
1. Use the template for the base of the cone and the five circles supplied by your teacher to make five cones. Tape but do not glue the surface to the base.

2. For each cone that you have constructed, measure the height and complete the table.

Radius of circle used for surface (cm)	6	8	10	12	14
Height of cone (cm)	5,2	7,4	9,5	12	14

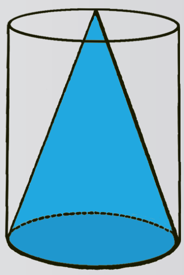


3. Use the template for the base of the cylinder and five rectangles to make five cylinders. The height of the rectangles must be equal to the heights of the cones measured in question 2 and the length of the rectangles should be at least 27 cm. In each case, only secure one of the bases of the cylinder.



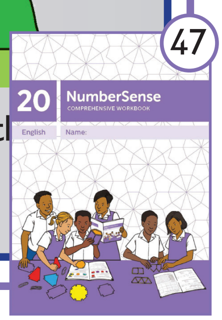
Note to teacher: the base should have a radius of 3 cm.

Each of the cones should fit snugly inside the corresponding cylinder like this:



To measure the volume of each cone, remove the base, fill the cone wit

pour the rice into a measuring jug and read off the volume.



4. Determine the volume of each cone and cylinder and complete the table.

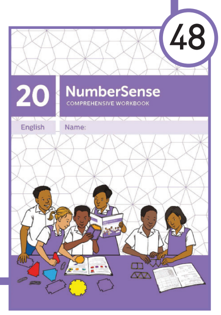
Height of cone and cylinder	Volume of cone (mℓ)	Volume of cylinder (mℓ)

5. For each pair of cones and cylinders, use a calculator to calculate the ratio of the volume of the cylinder to the volume of the cone.

ratio = volume of cylinder ÷ volume of cone

Height of cone and cylinder	Ratio of the volume of the cylinder to the volume of the cone

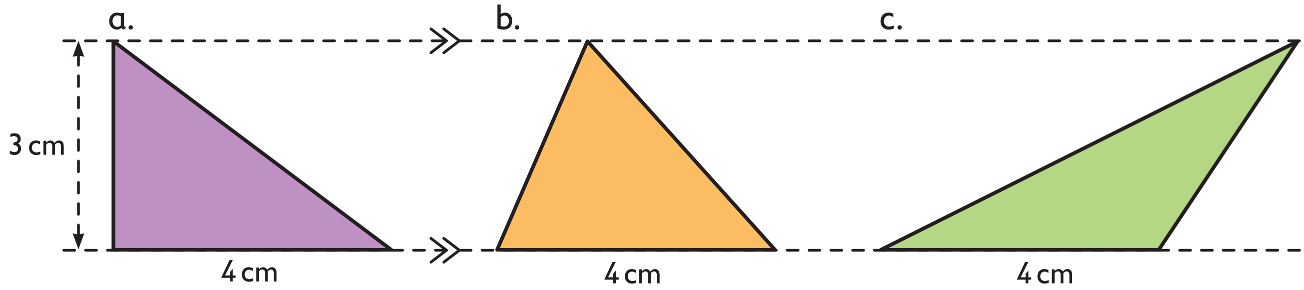
What do you notice? Discuss.



Application of perimeter, area and volume formulae

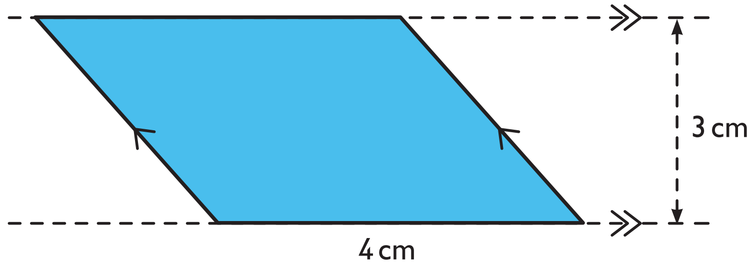
Area of triangles and parallelograms between parallel lines

1. Determine the area of the triangles. Take care to show your thinking.

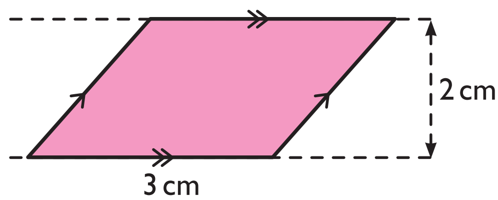


What do you notice?

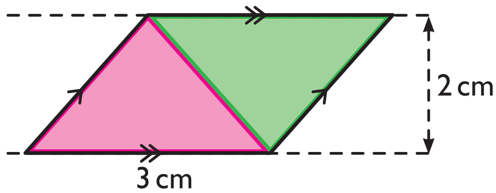
2. Determine the area of the parallelogram.



Determine the area of the parallelogram.



Sindi



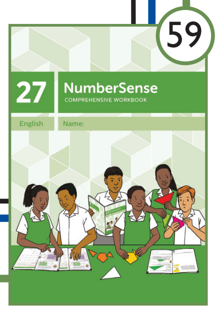
I add a diagonal to the parallelogram. We know that the diagonal bisects the parallelogram into two equal triangles. So,

area of parallelogram $= 2 \times$ *area of triangle*

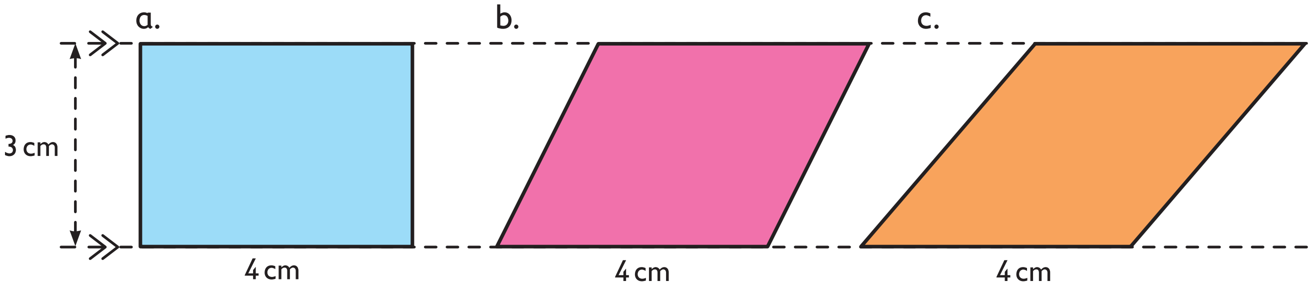
$$= 2 \times (\frac{1}{2} \times b \times h)$$

$$= 2 \times (\frac{1}{2} \times 3 \times 2)$$

$$= 6 \text{ cm}^2$$



3. Determine the area of the rectangle and the two parallelograms.
   Take care to show your thinking.



   What do you notice?

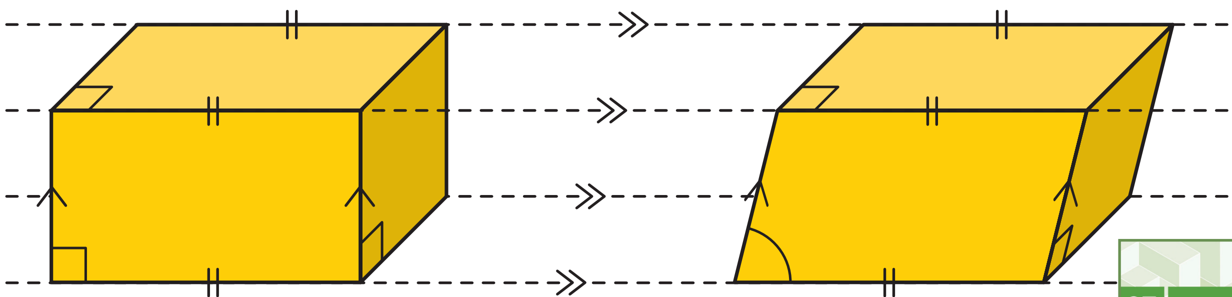
4. a. Are the perimeters of the triangles in question 1 the same?
      Explain why you say so.

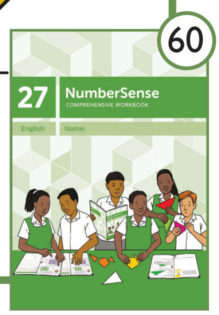
   b. Are the perimeters of the three parallelograms in question 3 the same?
      Explain why you say so.

Volume of objects between parallel surfaces

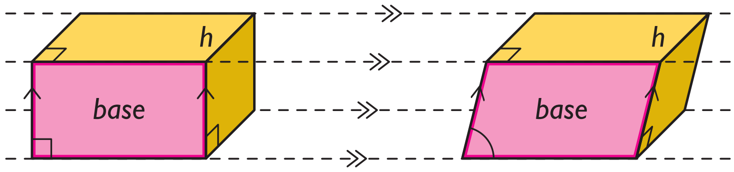
In the previous section we observed that for both triangles and parallelograms with bases that are the same length and which are between parallel lines the areas are equal. What happens in 3 dimensions?

1. Do the right rectangular prism and the oblique prism (the prism that has a face which is a parallelogram) have the same volume? Explain.





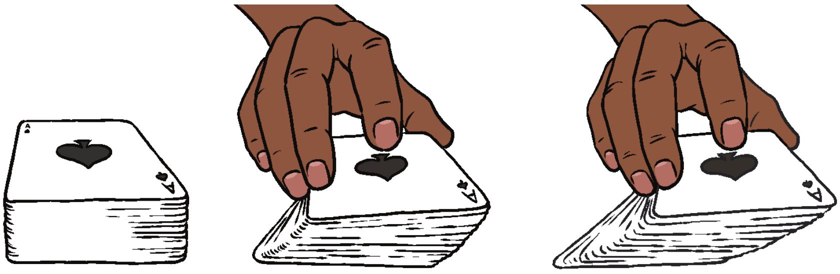
If we treat the faces that I have highlighted as bases then we



have two right prisms with these bases. As we have already shown, these bases are equal in area and since

volume of a prism = area of base × height

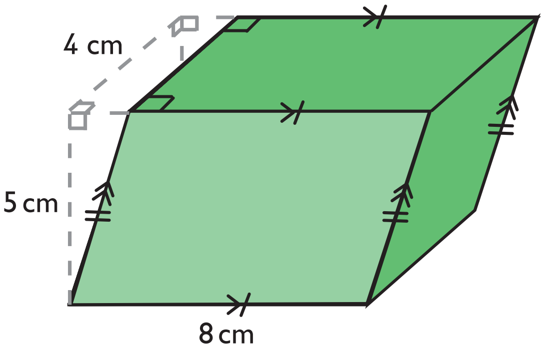
the two prisms have the same volume.



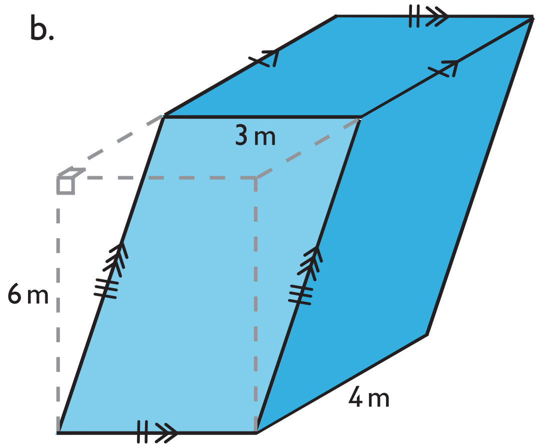
I think of it completely differently. Imagine a large pile of playing cards. I can change the shape of the pile by moving the cards around. The shape of the object will change but the volume (number of cards) does not!

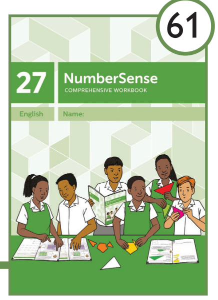
2. For each example, shade the face that you are using as your base and determine the volume of each prism.

a.



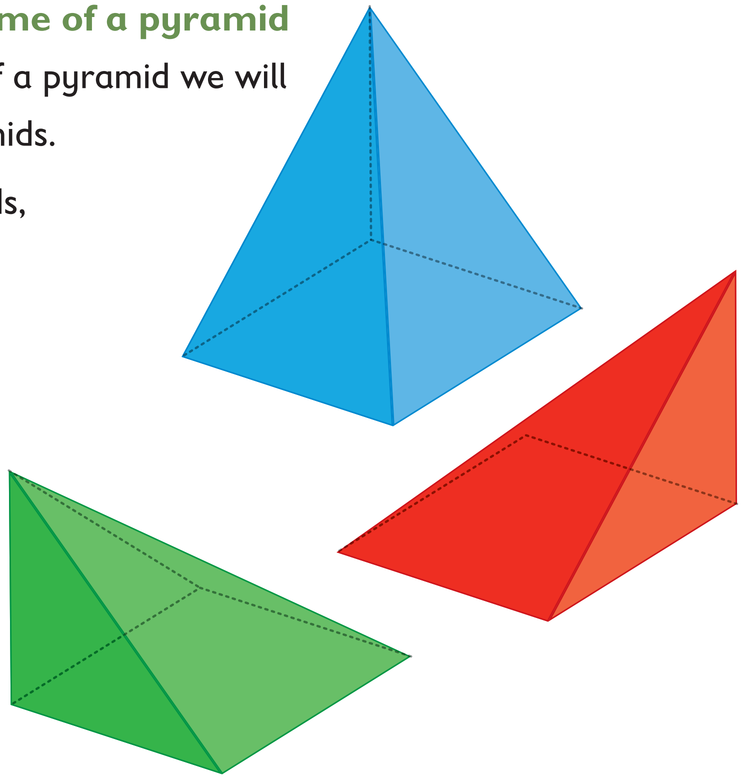
b.





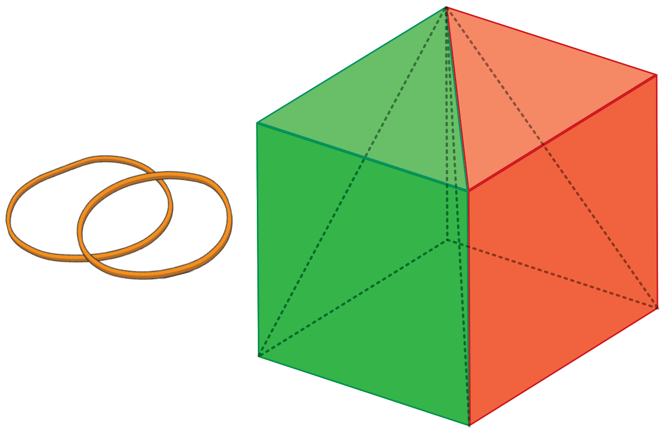
Developing a formula for the volume of a pyramid

To develop a formula for the volume of a pyramid we will first make a cube using identical pyramids.



1. Make three square-based pyramids, as follows:
   - Trace three copies of net 1 on page 64.
   - Stick each net onto a different coloured piece of light cardboard.
   - Cut out the three nets and build the pyramids.

2. Join the pyramids to make a cube. You should use rubber bands to hold the pyramids together.



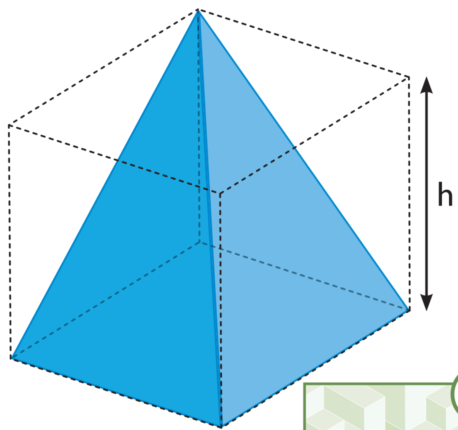
3. What fraction of the cube is the blue pyramid? Explain.

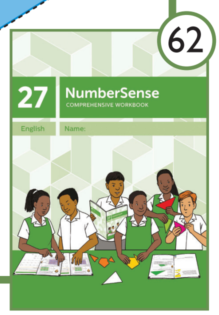
4. Use the formula:

   volume of prism = area of base × ⊥height,

   to justify that the volume of the pyramid is given by:

   volume of pyramid = $\frac{1}{3}$ × area of base × ⊥height

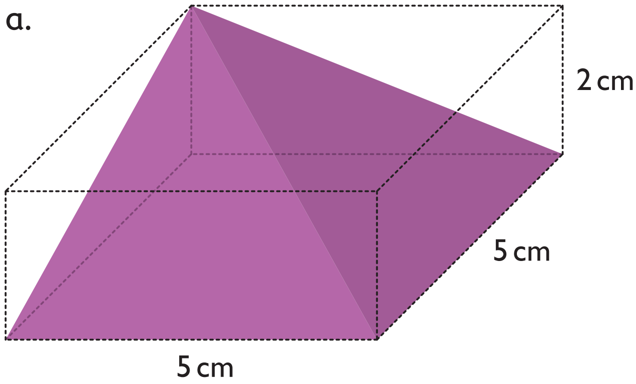




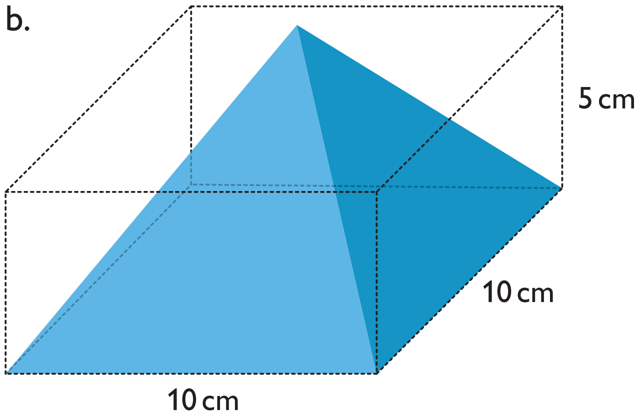
Measurement

5. Use the formula for the volume of a pyramid to determine the volume of each pyramid.

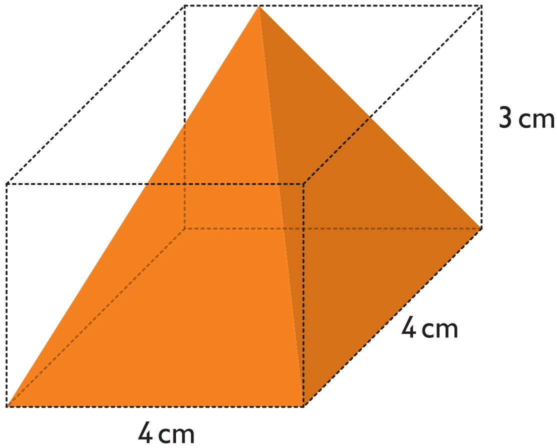
a.



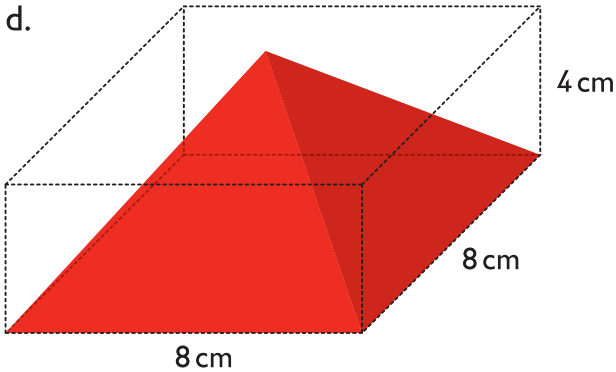
b.



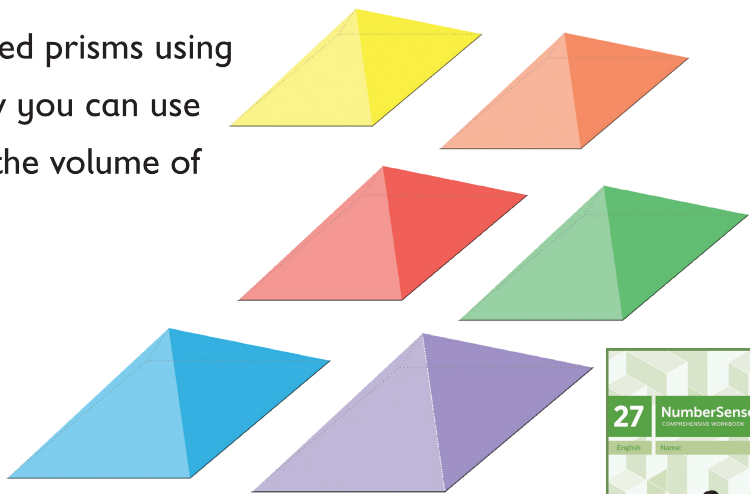
c.

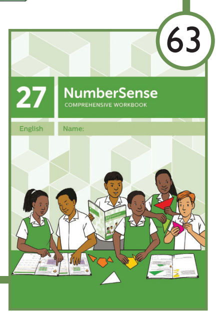


d.

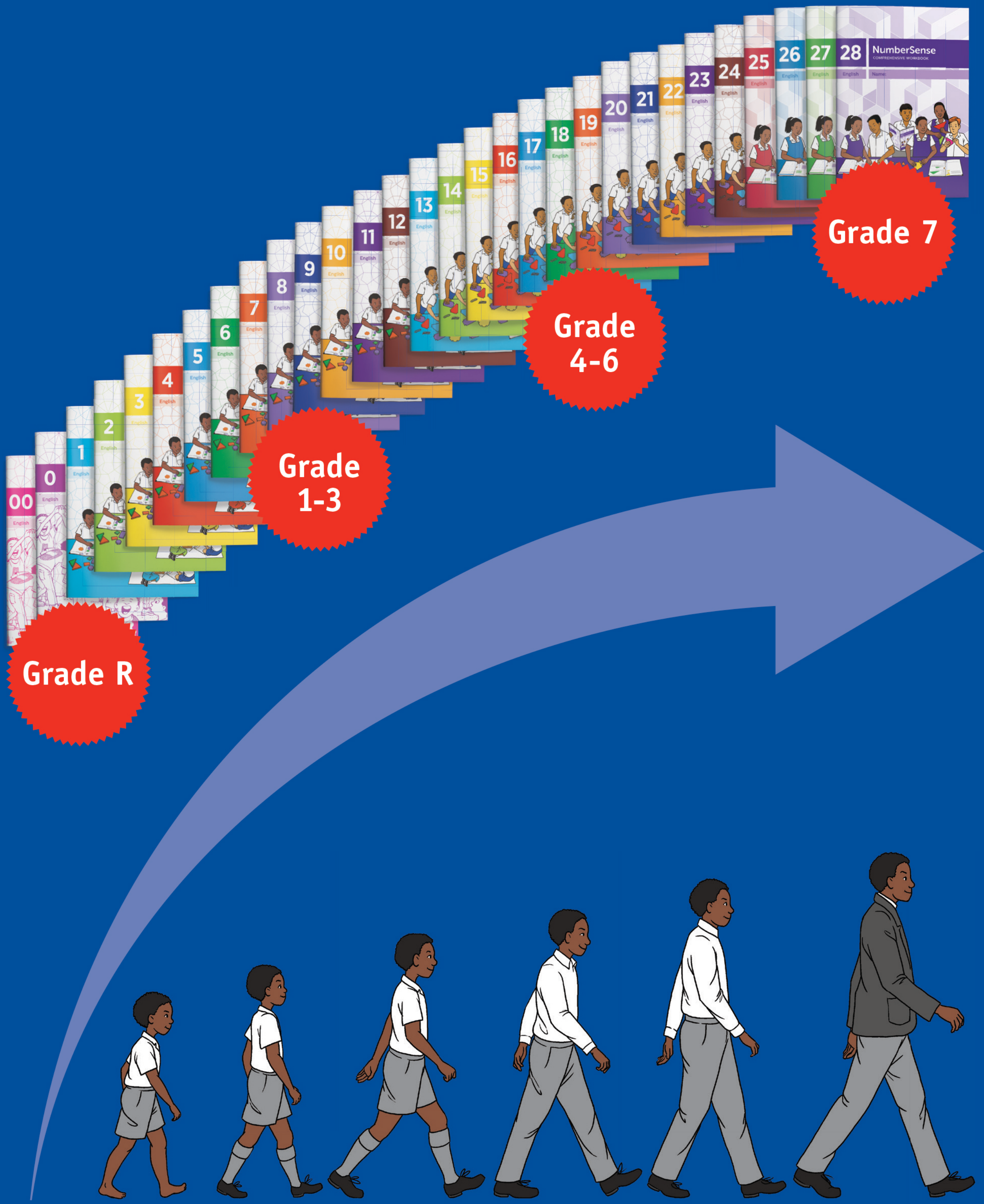


6. <u>Challenge</u>: Make six square-based prisms using net 2 on page 64 and show how you can use these to justify the formula for the volume of a pyramid.





NUMBERSENSE COMPREHENSIVE WORKBOOK JOURNEY



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