PROBLEMS AND INVESTIGATIONS

FIRST DRAFT
TEACHER GUIDE
CHAPTER 3.4

Brombacher & Associates

1. Sara has 123 marbles, Vusi has 76 marbles, and Sethu has 45 marbles, [illegible]?

2. Jan has 84 marbles, [illegible] 4 marbles [illegible]?

3. Suzie [illegible] 23 [illegible] 4 marbles [illegible]?

4. Mark has 34 marbles. He plays a game [illegible] Thandi [illegible] 12 marbles. How many marbles [illegible]?

5. Maria has 64 marbles. [illegible] Her father [illegible] 100 marbles [illegible]?

6. [illegible] 132 marbles, [illegible] 57 [illegible]. How many marbles [illegible]?

Problems and Investigations in the NumberSense Mathematics Programme

Exercises, problems and investigations

The different activities that take place in a mathematics classroom can typically be classified as an exercise, problem, or an investigation.

Exercises

Exercises or practice tasks are activities that involve well-defined procedures. Exercises are typically used for practising newly-learned skills.

Exercises are important in a mathematics classroom. Children need to practise and consolidate their understanding of skills, procedures, vocabulary and so on.

The danger is that mathematics is often reduced to exercises, with teachers teaching procedures and children practising them without much recourse to understanding.

Traditionally, the solving of word or story sums has been characterised as "problem solving". The issue is that these word or story sums can often be solved without even reading the text of the so-called problem. Furthermore, teachers teach strategies to determine the solution, strategies that do not rely on reading the text and/or understanding the situation. Strategies such as "circle the numbers, look for and circle the clue words, write a number sentence, solve it and underline the answer" In some instances, the relationship between the numbers in the text can be determined simply by looking at the section heading (e.g. multiplication).

If the child knows what to do, they are doing an exercise, not solving a problem.

A consequence of teaching mathematics by topic is that we see families of so-called problem types appearing at the end of sections or chapters, for example, "division problems," "multiplication problems," and "ratio and rate problems."

The NumberSense Mathematics Programme recognises the need for children to practise and consolidate their understanding of skills, procedures, vocabulary etc., and provides ample opportunities for children to practise using and applying the skills and procedures that they have developed.

Problems

A problem is a situation that requires resolution and for which the way of getting to the resolution in not immediately clear.

Problems are typically well stated and mostly have only a single solution. Children must, however, "make a plan" to solve the problem and that plan involves selecting an appropriate strategy to use. Children need to develop a range of problem-solving strategies that they can draw on when solving problems.

Problems also require the child to be able to reason and justify.

Over the last twenty or more years, the literature on teaching mathematics, as well as curriculum statements, have identified problem solving and reasoning as a priority in effective mathematics instruction. The reality, supported by research, is that children who are confident problem solvers are more confident about their ability to do mathematics in general.

Unfortunately, the shift to problem solving and reasoning has not taken hold to the extent that it should because teachers tend to be more concerned about preparing children for tests and examinations. Tests and examinations that typically focus on assessing procedural knowledge.

A challenge of teacher education (training) is to focus on what problem solving is and on the different ways in which problem solving can be used in the classroom.

Problems and problem solving are used in different ways in the mathematics education.

- Teaching *about* problem solving. In this case, problem solving is the subject of the programme or course. The study of problem solving is most typically a subject in teacher training courses and a field of research in mathematics education.
- Teaching *for* problem solving. Teaching for problem solving involves explicitly teaching problem solving skills or strategies that children can draw on when confronted with problems.
- Teaching *through* problem solving. Teaching through problem solving involves using problems to develop mathematical skills and concepts.

The NumberSense Mathematics Programme uses problems both to develop mathematical skills and concepts (teaching through problems), as well as to develop problem-solving strategies.

Investigations

Investigations, in contrast to problems, are more open-ended tasks that require children to apply mathematical concepts and skills creatively to complete them. Investigations encourage children to pose questions, develop conjectures, experiment, and draw conclusions based on their findings.

Investigations develop critical thinking, involve applying knowledge in unfamiliar situations, and require reasoning to justify findings. In addition, they develop communication skills as children often work in groups to complete an investigation.

The NumberSense Mathematics Programme uses investigations predominantly in geometry (space and shape) to explore properties of and relationships between geometric shapes and objects; in measurement as a context for developing an appreciation of the key issues involved in measuring (understanding what is being measured and how best to do so); and, in data handling to answer questions that require the collection, organisation and interpretation of data.

Summary

An exercise is a task where the child knows what is being asked and knows what to do.
A problem is a task where the child knows what is being asked but does not know what to do.
An investigation is a task where the child does not know what is being asked and does not know what to do.

Teaching mathematics through problems

Mathematics is a tool for solving problems.

Problems also provide a medium for teaching mathematics.

Children can add, subtract, multiply, divide, work with fractions, use formulae and solve equations long before these operations and concepts are formalised and long before they have mathematical vocabulary with which to describe what they are doing. When a mother gives her children some sweets and asks them to share the sweets equally amongst themselves, they can do so without describing what they are doing as dividing and without writing a mathematical expression to describe what they have done.

Living organisms are natural problem solvers. Consider a plant growing in the ground. If the root meets a stone, it grows around the stone. When an animal senses danger, it will run away and hide or change colour or attack the perceived danger. When a young baby is hungry, he/she will cry to get attention. Children who come to school know how to solve problems – what they do not know are the labels that adults use to describe what they are doing. This is particularly true in mathematics.

When we present a young child with some toy animals and a pile of counters and ask the child to share the counters equally between the animals, they will do so. Try it! You will be amazed. Young children have naturally efficient strategies for sharing the counters between the toy animals. Young children can solve this problem and problems like it long before they can count the counters in the pile, long before they know what it is to divide, and long before they can write a number sentence to summarise the problem situation and its solution.

Children come to school with an incredible capacity for solving problems in general. One only needs to watch children at play to realise how inventive and clever they are.

When we teach mathematics through problems, we present children with problem situations that they can make sense of and resolve. The organic response that they use is the mathematics we want them to learn more formally. In other words, we use a problem to provoke a response and that response is the mathematics we want to teach/develop. This approach is not limited to the basic operations - this approach applies throughout school mathematics.

The descriptions of classroom episodes on pages 13 and 19 illustrate this point.

Problems serve three important roles when teaching through problem solving (using problems as a pedagogical device):

- Carefully designed problems introduce and give meaning to mathematical concepts, including operations.
- Deliberately sequenced problems contribute to the development of age, developmental level and number-range appropriate calculation strategies.
- Solving problems that are 'sincerely problematic' allows children to experience mathematics as significant and relevant to the world in which they live.

The defining characteristics of problems used for teaching mathematics through problem are:

- The strategies or methods used to solve the problem(s) are not immediately obvious. That is, problems should require children to make a plan (see the discussion on problem solving-strategies).

- The problem(s) should provide children with the opportunity to apply and connect with previously developed mathematical understanding.
- The problem(s) should be novel. If the problem is one that the child has solved several times before and knows exactly what to do, then the problem is not a problem, it is simply an exercise.

Using problems to introduce children to the mathematics we want them to learn

In a classroom where we teach mathematics through problems, we use a carefully structured/designed sequence of problems to provoke the mathematics we want to teach. The responses of children are representations of "the mathematics" that we want children to develop.

We have a choice when teaching mathematics – particularly so in the early grades. Either we start the lesson and say, "Today we are going to learn about ratio. Ratio is defined as …". Or, we work through a sequence of problems that provoke predictable responses, responses that reveal an understanding of ratio and then give a name to the children's responses and say, "What we have been doing is described as sharing/dividing the biscuits in a ratio. A ratio is written as …".

The key difference in the approaches is that the "teaching mathematics through problems" approach" assumes that children have the capacity to make sense of situations. In addition, by making sense of situations children will "do" the mathematics organically. Furthermore, by "generating" the mathematics themselves, they will both experience it as more meaningful (less abstract) and with greater understanding.

The danger of this discussion is that the reader develops the impression that posing "word" problems is enough to help children learn mathematics. This is not the case. At the heart of the "teaching mathematics through problems approach" is the very deliberate design of the problems used (the science of teaching).

As critical as the deliberate design of the problems is, so too is the need for discussion and reflection. We learn by reflecting (reflective abstraction). In addition to using deliberately designed problems, the other critical role of the teacher is to facilitate reflection by children on what they have done. When we ask children to explain what they have done, to describe what they understand their friend(s) to have done, and to solve the problem using an approach/strategy that one of the other children has used and explained to the group, we cause children to reflect. It is through this reflection that learning takes place.

In the early grades, teachers first use problems to introduce children to the four basic operations. Along the learning journey there comes a time where children are familiar with the basic operations and can perform these without recourse to the problem(s) that gave rise to them. Problems are then used to introduce fractions and the ratio concept. And many years later, deliberately designed optimisation problems provoke the need to determine the turning point of a curve and differential calculus is introduced.

Problem types

Addition and subtraction are introduced through problems that involve:

- Changing the number of objects,
- Combining two or more sets of objects, and
- Comparing two or more sets of objects.

These different problem types support the development of the concept of addition and subtraction. By solving these problems, learners are not only exposed to different situations that involve addition and subtraction, but also to the interrelationship between these operations.

Division is introduced through problems that involve:

- Sharing objects, and
- Grouping objects.

Sharing and grouping problems both introduce the concept of division. However, by their structure they develop different yet complementary understandings of what it means to divide, thereby developing a range of different strategies for performing division.

Multiplication is introduced through problems that involve:

- Groups of objects.

These problems involve groups of objects with the same number of objects in each group and the need to determine the total number of objects. When the groups of objects in the problem are arranged in grids or arrays, the problem also supports an understanding of the commutative nature of multiplication.

The fraction concept is introduced through problems that involve:

- Sharing objects with remainders that can be partitioned.

Sharing problems with remainders that can be partitioned introduces the concept of a fraction as a part of a whole. Carefully structured problems not only reveal the fraction as a part of a whole, but also support the development of a conceptual basis for comparing fractions, creating equivalent fractions, and calculating with fractions (see Appendix A for an illustration of how the fraction concept is taught through problems in the NumberSense Mathematics Programme).

Problem types used to introduce addition and subtraction

	(✓ ± ✓ = ?)	(✓ ± ? = ✓)	(? ± ✓ = ✓)
change problems	Ben has 8 toffees. His father gives him 6 more toffees. How many toffees does Ben have now?	Ben has 8 toffees. His father gives him some more toffees. He now has 12 toffees. How many toffees did his father give him?	Ben has some toffees. His father gives him 6 more toffees. He now has 14 toffees. How many toffees did he begin with?
	Ben has 8 toffees. He eats 2 of his toffees. How many toffees does he have left over?	Ben has 8 toffees. He eats some of his toffees. He now has 5 toffees. How many toffees did he eat?	Ben has some toffees. He eats 3 of his toffees. He now has 5 toffees. How many toffees did he start with?

	(✓ + ✓ = ?)	(✓ + ? = ✓ or ? + ✓ = ✓)
combine problems	There are 4 boys and 5 girls in a class. How many children are there in the class altogether?	There are 14 children in a class. 5 of them are boys. The rest are girls. How many girls are there in the class?
	Fundi makes 5 cookies. Jan makes 8 cookies. They put their cookies together in a box. How many cookies are there in the box?	Altogether, Fundi and Jan made 14 cookies. If Fundi made 5 cookies, how many cookies did Jan make?

compare problems	Vusi has 8 apples. Sara has 3 apples. How many more apples does Sara need to have the same number of apples as Vusi?
	Vusi has 12 apples and Sara has 7 apples. How many apples must Vusi give away to have the same number of apples as Sara?

The following combine problem could be thought of as an addition problem. "Altogether, Fundi and Jan made 14 cookies. If Fundi made 5 cookies, how many cookies did Jan make?" i.e. 5 + ? = 14, asked what must be added to 5 to get 14. Children should recognise that to answer the question they must take 5 cookies away from the total.

The number sizes that are used in these problems should be changed to match the developmental level of the children. This in turn supports the development of age appropriate ways of recording thinking.

Problem types used to introduce division

Sharing and grouping are the two problem types used to provoke the development of division.

In a **sharing** problem, the natural response is to:

- Give each child some (the same number) objects.
- Check if objects remain.
- Continue in this way until there are no objects left over.

This thinking is what is at the heart of a typical division algorithm.

sharing problems

Four friends share 12 sweets equally between them. How many sweets does each friend get?

In this **sharing** problem, the child might:

- Draw the four children and start giving each child 2 sweets (using 8 sweets altogether).
- Notice that there are sweets left over and give each child another sweet until there are no sweets left over.
- Count the sweets given to each of the children and conclude that each child got 3 sweets.

In a **grouping** problem, the natural response is to:

- Remove sets of objects until no objects remain.
- Count the number of sets.

This is to think about division as repeated subtraction or addition.

The actions and thought processes of the child solving sharing and grouping problems are quite different. These differences contribute to what will one day be a richer understanding of what it means to divide.

Notice how these two problems have the same mathematical structure i.e. $12 \div 4 = \square$ and yet each produces completely different reactions from the child.

grouping problems

> A farmer has 12 potatoes. He puts 4 potatoes in a packet. How many packets can he fill?

In this **grouping** problem, the child might:

- Draw 12 potatoes.
- Draw circles ("packets") around 4 potatoes at a time until there are no potatoes left.
- Count the "packets" and conclude that the farmer can fill 3 packets.

It is very important that from the start, children are exposed to sharing and grouping problems with remainders. It is important, firstly, because we do not want children to think that there are no remainders in a division situation. Secondly, sharing with remainders will be used to introduce the fraction concept at a later stage. Finally, through carefully structured problems involving remainders, we force children to reason about the solution to the problem.

There are three different remainder situations:

- Situations where the remainder does not have an impact:

> A farmer has 15 apples. He puts 6 apples in a packet. How many packets can he fill?

The farmer can fill two packets and there are three apples left over. There is no impact on the number of packets that the farmer can fill.

- Situations where the remainder has an impact:

> A farmer has 15 eggs. He packs the eggs into boxes with 6 eggs in a box. How many boxes does he need to transport all the eggs to the market?

The farmer fills the first two boxes and still has three eggs left over. He will need another box to transport those eggs to the market.

- Situations where the purpose of the remainder is to provoke the development of the fraction concept:

> Masixole and Sara share five chocolate bars equally. Show them how to do it.

In this case, we expect the child to share the remaining chocolate bars between the two children, creating equal parts of a whole (see the sequence of problems used to develop the fraction concept on page 20).

Problem types used to introduce multiplication

Groups are used to introduce multiplication.

	repeated addition	grids and arrays
problems involving groups	Angela eats 4 slices of bread per day. How many slices of bread does she eat in 5 days?	There are 5 rows of mealies. There are 4 mealie plants in each row. How many plants are there altogether?

The two problems involving groups illustrate situations that provoke multiplication-like responses. These problems have the same mathematical structure, namely $5 \times 4 = \square$. However, the way in which the problems have been stated produces different reactions from the child.

The first problem can be thought of as repeated addition. The child solving the problem might think about (or draw) 4 slices of bread for each of the 5 days and determine the number of slices of bread eaten by adding $4 + 4 + 4 + 4 + 4 = 20$.

The second problem is a grid/array-like situation. The child solving the problem might think about (or draw) the 5 rows of plants with 4 plants in every row and end up with an image of a neatly organised grid/array. The grid/array will help the child to realise that they could calculate the number of plants by counting either $4 + 4 + 4 + 4 + 4 = 20$ or $5 + 5 + 5 + 5 = 20$. This encourages the realisation that five lots of 4 is the same as 4 lots of five. Mathematically $5 \times 4 = 4 \times 5$ is the so-called commutative property of multiplication.

The actions and thought processes of the child solving these problems are quite different. These different processes contribute to what one day will be a richer understanding of what it means to multiply.

Problem types used to introduce fractions

In the same way as there are carefully structured problem types to introduce addition, subtraction, division and multiplication, there are also carefully structured problems to introduce fractions. These problems follow on quite naturally from the sharing problems that are used to introduce division. For a detailed illustration of how a carefully structured sequence of problems is used to develop the fraction concept, refer to the illustrations on pages 20 to 32.

Ms Sibusa's fraction lesson

Working with a group of Grade 2s (7 and 8 year-olds), Ms Sibusa wanted to start introducing the children to the fraction concept. Ms Sibusa chose a well-designed problem and started, 'Today's problem deals with chocolate. Do you like chocolate? Really? What is your favourite chocolate bar?' She led some discussion about chocolate bars to get the children involved in the story and to make it meaningful for the children. She also had the discussion to make sure that the idea of a chocolate bar (as a rectangular shape) was clear in the minds of the children. Next, she said, 'In today's problem there are two children, Yusuf and Ben. They want to share three chocolate bars equally so that there is no chocolate left over. Can you show them how they can do that?'

The group of children set to work. Because children in this class were used to working on problems, they knew what they were expected to do. They tried to make sense of the problem and, because the problem was unfamiliar to them, many drew a picture of the situation.

Masixole drew two faces and three chocolate bars. Next, he drew a line from the first chocolate bar to the first face and then a line from the second bar to the second face. Then he paused for a while and thought about what to do with the remaining (third) chocolate bar. After a while he drew a line through the third bar of chocolate (as if to cross it out) and drew a small piece of chocolate next to each face.

After many of the children had grappled with the problem for a while, Ms Sibusa selected three different children to show and describe their solution strategies to the others. When it was Masixole's turn, he pointed at his picture and said, 'This is Yusuf and this is Ben. First, I drew the three bars of chocolate, and then,' he said, pointing at the lines, 'I gave one bar to Ben and one bar to Yusuf. Then there was one bar left over. I thought about this for a while and then decided to cut this bar into two pieces and to give Yusuf and Ben one piece each.'

Ms Sibusa asked Ben, 'Why did you cut the bar into two pieces?' and he responded, 'Because there were two children.' Ms Sibusa asked the other children in the group if they understood what Masixole and the others had done. After some discussion that included the children comparing their solutions, she posed two more problems. 'Jan, Sarah and Ben want to share four bars of chocolate equally so that there is no chocolate left over. Can you show them how they can do that?' and, 'Fundi, Jan, Sarah and Ben want to share five bars of chocolate equally so that there is no chocolate left over. Can you show them how they can do that?'

The children worked on the problems and the teacher observed them, asking individual children to explain to her what they were doing. Occasionally she made a suggestion to an individual child, or asked some questions that forced the child to think about what the problem was asking. Ms Sibusa noticed that Verencia drew careful representations of the problems. In particular, she noticed that Verencia's pieces from the leftover bar (which she gave to the children in each problem) were getting smaller from the first problem to the last. Ms Sibusa made a note to herself that she would ask Verencia to explain to the group why she had drawn these pieces as getting smaller and smaller.

After a while, Ms Sibusa once again asked carefully selected children to explain their solutions to the group. Ms Sibusa led a discussion and, when rounding off the discussion, asked the children to summarise what they had done. The group concluded that they have given each child in the problem a bar of chocolate and if there was a leftover bar, they would cut it up into pieces and give each child a piece. Ms Sibusa asked, 'Do you just cut the bar up into any number of pieces that you wish?' 'No,' said the children, 'You cut the bar into as many pieces as there are children. If there are four children you cut the bar into four pieces.' Ms Sibusa asked, 'So what if there are six children and one leftover bar of chocolate?' 'Then,' said one of the children, 'you cut the leftover bar into six pieces.'

Finally, Ms Sibusa asked Verencia, 'I noticed in your pictures that the pieces were getting smaller. Why was that?' Verencia responded, 'As more and more children had to share the leftover bar of chocolate, the pieces got smaller. The more children you share a bar of chocolate with the less each one gets.'

In just this one problem episode, Ms Sibusa, using a well-structured series of problems, had established the notion that a whole can be broken into any number of pieces/parts – in other words, fractions. The structure of the problems had also enabled Verencia to realise, in effect, that one half is larger than one third and that one third is larger than one fourth.

Using problems to support the development of age, grade and number range appropriate ways of recording thinking

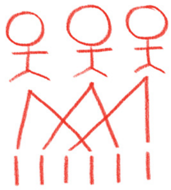
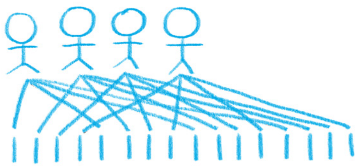
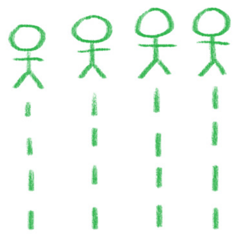
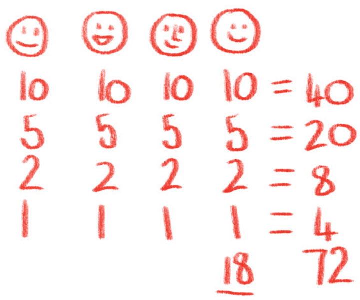
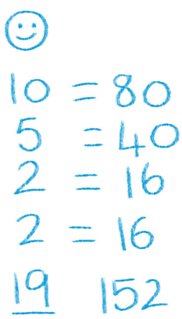
In a classroom where we teach mathematics through problems, we also use the sequence of carefully designed problems to introduce and support the development of age, grade and number range appropriate ways for children to record their thinking.

In a classroom where we teach mathematics through problems and expect children to be able to reason about what they are doing, it is important that children are encouraged to record their thinking in a way that makes sense to them. That said, it is also important that children record their thinking in ways that are appropriate to their age, their grade, and the number range in which they are working. Just as we do not expect children in the early grades to write number sentences, we would not expect children in later grades to still be drawing detailed representations of the problem they are working on.

Figure 1 illustrates a progression of age, grade and number range appropriate ways that children may use to record their thinking. Typically, children will progress through most of these stages for each new mathematical concept. As children get older, they may not have to rely on the early physical modelling approach. The progression through the sequence of age, grade and number range appropriate ways of children recording their thinking is orchestrated in large part by the teacher. As the teacher increases the number range of the problems, children need to become more efficient in recording their thinking which, in turn, leads to the use of mathematical notation and the associated writing conventions.

Figure 1:

Age, grade and number range appropriate ways of recording thinking

Problem	**Age, grade and number range appropriate strategies**	**Discussion**
The teacher places three dolls and 18 counters on the floor and asks a child: "Please share these counters equally between the three dolls."	Physical modelling 	Even before children can count to eighteen or read and write the number 18, they are able to make sense of this situation and solve the problem through physical modelling.
Three children share 6 sweets equally. How many sweets does each child get? Four children share 16 sweets equally. How many sweets does each child get?	Primitive drawings   Sophisticated drawing 	A simple (primitive) drawing works well for the first problem. Each child and each sweet is visible and it is easy to see what happens to each sweet. The same primitive drawing does not work for the second problem; there are too many sweets and it is hard to see how many sweets each child received. By consciously increasing the number of sweets in the problem, the teacher has provoked the children to use a more sophisticated strategy to solve the problem and record their thinking.
Four children share 72 sweets equally. How many sweets does each child get?	Primitive number strategy 	By increasing the number range further, the teacher has ensured that even a sophisticated drawing is inefficient and children begin using numbers to represent the sweets (which are now invisible). Notice how the child's solution uses the knowledge of groups and doubling and halving which should have been developed in the mental arithmetic work.
Eight children share 152 sweets equally. How many sweets does each child get?	Sophisticated number strategy 	With time, and as the number range increases even further, children will develop more sophisticated number strategies. In this illustration the child no longer draws all the children. Much of the detail is no longer visible.

The role of the teacher in supporting children to develop more efficient ways of recording their thinking.

One way in which teachers contribute to children developing increasingly efficient ways of recording their thinking is by managing the size of the numbers in the problem. As the number range of the problem increases, children must develop more sophisticated ways of recording their thinking.

Another very important role that the teacher can play, is in the way that they manage the discussion.

When teachers set a problem for children to solve, it is important that the teacher allows enough time for children to work on the problem independently. While the children are working independently, the teacher observes the children and, in her mind, classifies each child's approach in terms of its level of sophistication.

After the children have had enough time and most of them have solved the problem, the teacher then manages a discussion of the different ways in which children have recorded their thinking. Typically, she will ask a child whose recording is not very sophisticated (in terms of the age of the child) to explain what they did so that the other children in the group who were struggling to make sense of the problem can be inspired to use this approach. She will also select one or more of the children with more sophisticated approaches to explain what they did so that, in turn, those using less sophisticated ways of recording their work will be inspired to use a more sophisticated (efficient) approach. The teacher may also ask a child who made a mistake to explain their thinking and record their thinking so that the group can discuss why that approach did not work – there is much to be gained from a discussion of mistakes.

By asking different children whose ways of recording their thinking varies in sophistication to explain their thinking to the others, the teacher exposes children to increasingly sophisticated ways of thinking and of recording that thinking. This exposure to more sophisticated approaches, coupled with the increasing demand of the problems (resulting from the increased number ranges etc.) encourages children to develop increasingly more efficient ways of solving the problem and recording their thinking.

Managing the problem-solving component of the teacher-led activity

Solving a problem, or series of problems, is part of the everyday routine of the mathematics lesson in all grades.

The problem-solving activity typically involves the following stages:

- Before the lesson:
  - The teacher plans the problem to be used.
- During the lesson:
  - The teacher poses the problem,
  - Children work on the problem while the teacher monitors their progress, and
  - The teacher manages a discussion of the solutions (including mistakes) made by the children.

Planning the problem

The teacher develops the problem to be posed in the teacher-led activity. Mindful of the mathematics that she wants the problem to provoke (see problem types earlier pages 9 to 12), she chooses a problem. If the children are working in the NumberSense Workbooks, her choice of problem will also be guided by the problem that the children will be working on, on the workbook page of the day. She will match her problem to the problem type on the page. The number range of the problem she develops will be a little higher than the number range of the problems in the workbook. This is because with the teacher's help children can work in a higher number range than they are able to do independently.

Planning the problem for the teacher-led component of the lesson includes the teacher anticipating how children will tackle the problem and how they may record their thinking.

Posing the problem

The teacher introduces the problem as a story with a question that needs resolution.

She will ask follow-up questions to make sure that the children have understood the problem.

Finally, she will encourage the children to work on the problem in their exercise books, encouraging them to do so in a way that makes sense to them, and to record their thinking in a way that will enable them to explain their thinking to the other children.

Children working on the problem

The children work independently to solve the problem in their exercise books. As the children are working, the teacher looks over their shoulders to observe how they are recording their thinking and identifies the children she will invite to explain their thinking to the other children in the group (see the discussion on this in the previous section).

Reflection

The teacher identifies and invites the specific children she has identified (including children who have made mistakes) to describe how they solved the problem and how they recorded their thinking.

The teacher facilitates reflection by asking questions such as:

- Do you understand what he did?
  - If yes, "Please explain what he said in your own words."
  - If no, "Listen carefully, we will ask him to explain again."
- "How is what he did like what you did and/or different from what you did?"

The reflection is the most important part of the problem-solving activity. Without the reflection children are unlikely to benefit (learn) from the problem-solving episode.

Mr Nkhosi's ratio lesson

Working with a group of Grade 3s (8 and 9 year-olds), Mr Nkhosi set the scene. He said, 'In today's problem there are two dogs. A small dog and a large dog. Every time that the small dog gets one biscuit, the large dog gets two. If there are twelve biscuits altogether, how many biscuits will each dog get?'

The group of children set to work.

Ben drew a small circle and a large circle and then drew stripes, counting as he did so: 1 (under the small circle); 2, 3 (under the large circle); 4 (under the small circle); 5, 6, (under the large circle) and so on until he ended on 11, 12 (under the large circle). Next, he counted the stripes under each circle and concluded that the small dog would get 4 biscuits and the large dog would get 8 biscuits.

Fundi drew a small circle and a large circle. Instead of drawing stripes, she wrote the numeral 1 under the small circle and the numeral 2 under the large circle. On the next line she again wrote a 1 under the small circle and a 2 under the large circle and repeated this one more time. Then she paused and counted up as follows: starting under the large circle, she said, 'two, four, six' and, switching to the numbers under the small circle, she continued, 'seven, eight, nine'. Realising that she had not reached twelve yet, she wrote another 1 under the small circle, and a 2 under the large circle. She then counted again and, realising that she had reached 12, she added up the numbers under each circle and concluded that the small dog would get 4 biscuits and the large dog would get 8 biscuits.

Mr Nkhosi observed the children working. After some time, he invited Ben to show the other children how he had solved the problem. He asked Fundi to do so as well. Once they had finished explaining, Mr Nkhosi asked the other children if they understood what Ben and Fundi had done.

After some discussion, that included the children comparing their solutions to those of Ben and Fundi, Mr Nkhosi posed a new problem. He said, 'In my next problem there are two dogs. A small dog and a large dog. Every time that the small dog gets one biscuit the large dog gets three. If there are twenty biscuits altogether, how many biscuits will each dog get?'

Again, the group of children set to work and Mr Nkhosi observed them solving the problem. After some time, Mr Nkhosi asked two of the children to show their solutions to the others. He asked Sara to show her solution first. Sara had struggled on the first problem, but now, inspired by Ben's solution, she solved the problem using 'Ben's method'. Frank who had solved the first problem in a way that was similar to Ben's solution demonstrated his solution strategy. This time Frank used numbers instead of stripes – much like Fundi – and explained that he switched to 'Fundi's method' to avoid having to draw twenty stripes. Again, Mr Nkhosi asked the other children in the group if they had understood what Sara and Frank had done.

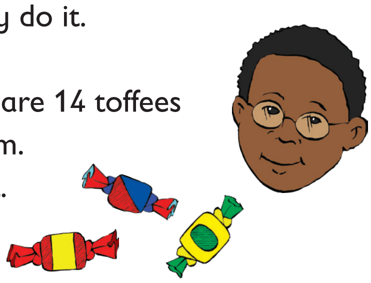
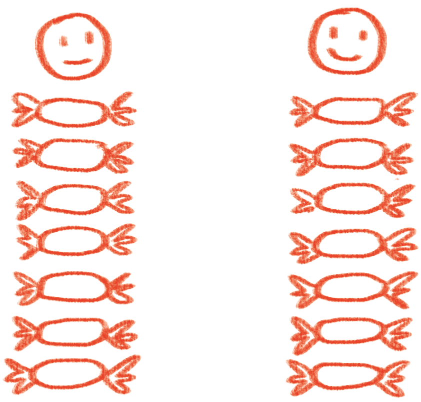
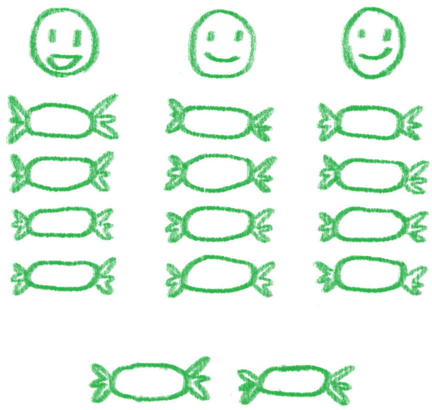
After some discussion, that included the children comparing their solutions to those of Sara and Frank, Mr Nkhosi posed a new problem. He said, 'In my next problem there are three dogs. A small dog, a medium dog and a large dog. Every time that the small dog gets one biscuit, the medium dog gets two biscuits, and the large dog gets four. If there are fifty-six biscuits altogether, how many biscuits will each dog get?'

The group of children set to work with different children solving the problem in different ways. Some continued to use 'Ben's method' even though it was proving a little less efficient with the larger number of biscuits. Some children used 'Fundi's method'. Some children noticed that after giving the small dog one biscuit, the medium dog two biscuits and the large dog four biscuits they had given away seven of the fifty-six biscuits. Instead of continuing to 'hand out biscuits', one, two and four at a time, they simply divided fifty-six by seven and concluded that the small dog would get eight biscuits. The medium dog would get eight times two – sixteen biscuits. The large dog would therefore get thirty-two biscuits.

Mr Nkhosi had, in effect, asked Grade 3 children to divide 56 in the ratio 1 : 2 : 4 – and they had done so with confidence.

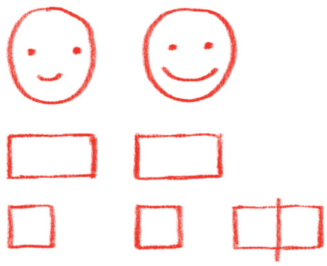
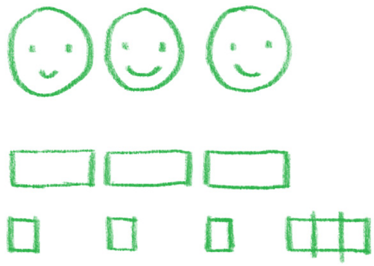
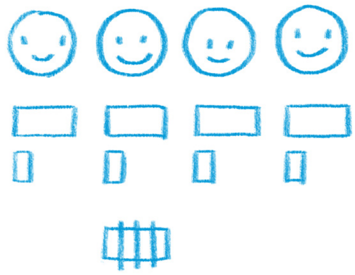
Illustration of how a carefully designed sequence of problems can be used to develop the fraction concept.

1. Problems that involve sharing with remainders

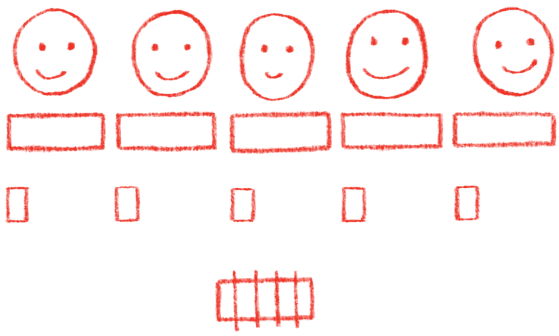
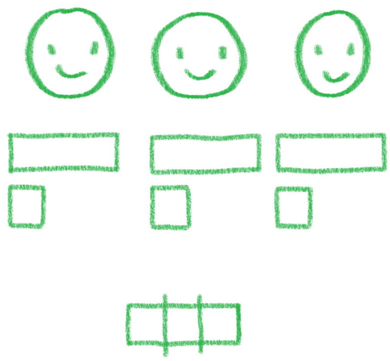
NumberSense Workbook problem(s)	Comments
Sara and Fundi share 14 toffees equally between them. Show how they do it. Jan, Ben and Yusuf share 14 toffees equally between them. Show how they do it. 	These problems develop an understanding of equal sharing (division). Right from the start, the equal sharing problems have left overs (remainders). This is to avoid the development of the misconstruction that division cannot have a remainder. At this stage, the objects being shared are typically objects that cannot be partitioned (easily cut into equal pieces).
Anticipated response(s)	
	

2. Problems that involve sharing with remainders that can be partitioned

NumberSense Workbook problem(s)	Comments
Fundi and Yusuf want to share 3 chocolate bars equally. Show them how to do it. Jan, Sara and Ben want to share 4 chocolate bars equally. Show them how to do it. Yusuf, Ben, Jan and Fundi want to share 5 chocolate bars equally. Show them how to do it. 	These problems involve objects that can easily be partitioned so that the remainder can be cut into pieces – parts of a whole. The problems deliberately have the same structure as the sharing with remainder problems so that children feel confident in making a start. The problems draw attention to the notion that a whole can be cut into any number of pieces (two pieces, three pieces, four and five pieces and so on). By asking "Show them how to do it" rather than "How much does each child get" the problem avoids the expectation of a numerical solution. This encourages children to make sense of the problem by, for example, drawing a picture, and focuses the children's attention on the existence of a part of a whole.

Anticipated response(s)		
		

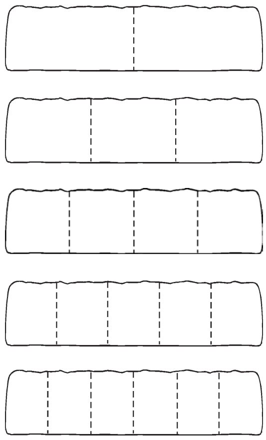
3. Problems that involve sharing with remainders that can be partitioned and compared

NumberSense Workbook problem(s)	Comments
Jan, Sara, Ben, Vusi and Fundi share 6 chocolate bars equally. Show them how to do it. Yusuf, Sindi and Lebo share 4 chocolate bars equally. Show them how to do it. Who get more chocolate, Fundi or Sindi? Explain.	The expectation is that the children respond conceptually to this problem. Children intuitively recognise that the more children who share a chocolate bar, the smaller the piece that each child will get.
Anticipated response(s)	
	

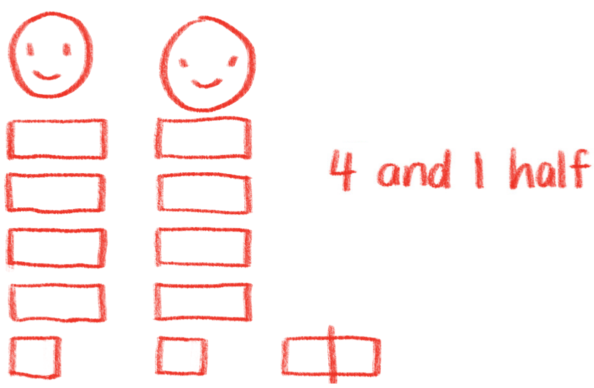
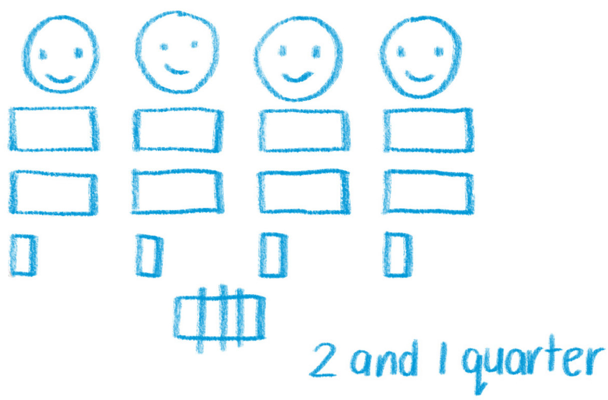
Discussion of the earlier problems
Children often experience the fraction concept as difficult; it is for this reason that the sequencing and nature of problems/tasks needs to be thoughtfully considered. Notice how in the sequence so far the problems: • Introduce the need for a part of a whole (a fraction) by using a sharing problem with a remainder which can be partitioned. • Immediately have different numbers of children sharing the remainder to make it clear that the remainder can be partitioned into any number of pieces. This is to avoid the misconception that easily arises when teachers introduce fractions one fraction at a time, namely that there are only two fractions: halves (big pieces) and quarters (small pieces).

- Draw attention to the fact that parts of a whole (fractions) can be compared by recognising that as the number of people sharing the remainder increases, so the part of the whole that each person receives gets smaller.
- Delay the introduction of the word fraction, fraction names, and fraction notation (Piaget's so-called social knowledge) until children have a solid conceptual understanding of fractions.

4. The introduction of fraction names

Information box	Comments
If a chocolate bar is cut into: • two equal parts, we call them **halves** • three equal parts, we call them **thirds** • four equal parts, we call them **fourths** • five equal parts, we call them **fifths** • six equal parts, we call them **sixths** 	The use of fraction names or words (rather than notation) during the early development of the fraction concept is very deliberate. The words convey the meaning of the fraction better than the notation does. The notation will be introduced later.

5. Problems that involve rephrasing of sharing problems with fractional remainders to force the use of fraction names

NumberSense Workbook problem(s)	Comments
2 friends share 9 bars of chocolate equally. How much chocolate will each one get? 4 friends share 9 bars of chocolate equally. How much chocolate will each one get?	Notice that the question has shifted from "Show them how to do it" to "How much does each child get?". Since the language for fractions has been introduced, the expectation is that children can now describe their solutions using more formal language to describe the part.
Anticipated response(s)	
	

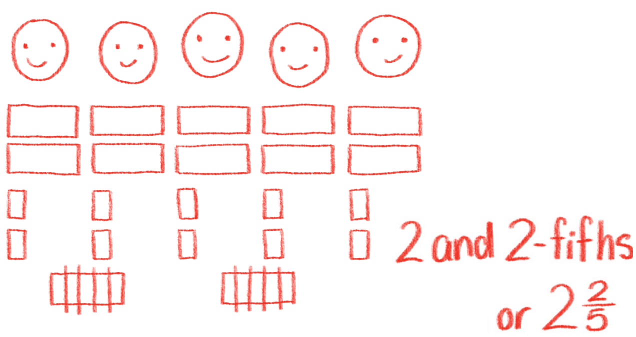
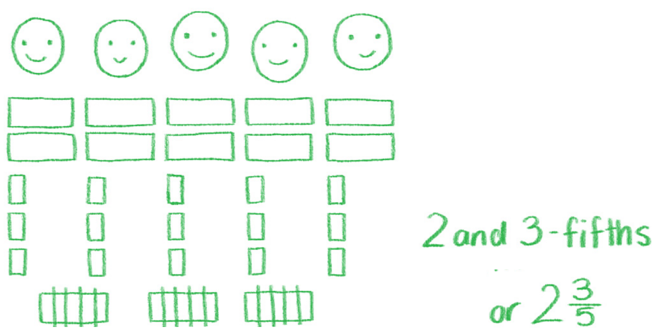
6. The introduction of fraction notation

Information box	Comments
A short way of writing one half is $\frac{1}{2}$ A short way of writing one third is $\frac{1}{3}$ A short way of writing one quarter is $\frac{1}{4}$ A short way of writing one fifth is $\frac{1}{5}$	It is important that while the notation has been introduced, teachers and children still use the fraction names that were previously introduced. For example, $\frac{1}{3}$ should be read as "1-third" and not "1 over 3"; $\frac{1}{5}$ should be read as "1-fifth" and not "1 over 5". Problems such as the illustrative problems previously discussed, can be asked again with the expectation that learners will now describe their solution using the fraction notation. While the use of fraction notation should be encouraged at this stage, teachers should allow some flexibility between the fraction words and fraction notation.

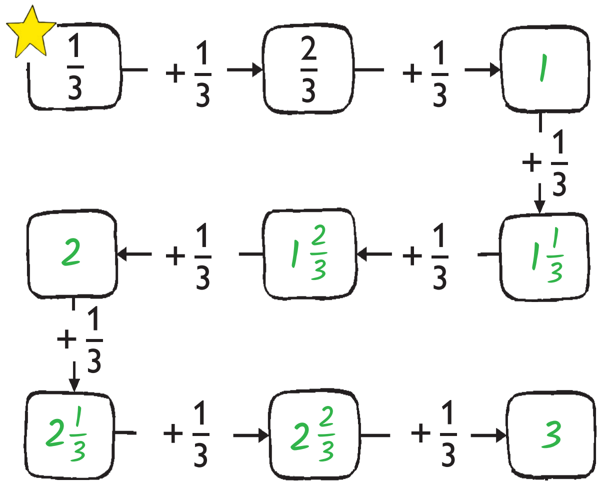
7. The introduction of fraction notation for non-unitary fractions

Information box	Comments
A short way of writing one third is $\frac{1}{3}$ A short way of writing two thirds is $\frac{2}{3}$	As children gain confidence in working with fractions, some of the problems will start to introduce non-unitary fractions and the corresponding notation is needed.

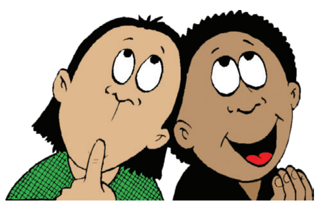
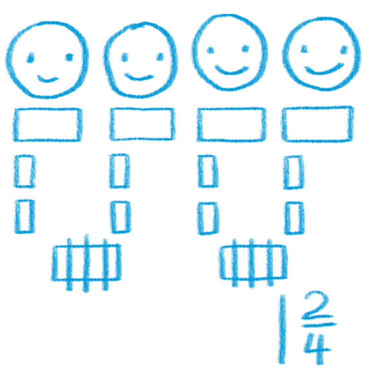
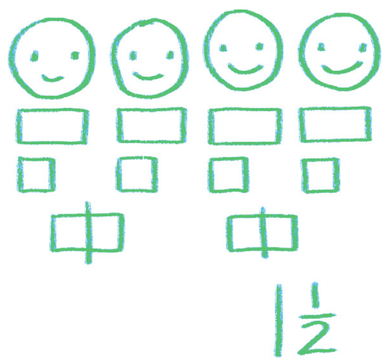
8. Problems that involve non-unitary fractions

NumberSense Workbook problem(s)	Comments
Five friends share 12 chocolate bars equally. How must they do it? Five friends share 13 chocolate bars equally. How must they do it?	In these problems there is more than one chocolate bar left over, which leads to solutions involving non-unitary fractions. For these problems it is important that the number of chocolate bars left over is not a factor of the number of children sharing the chocolate. This would introduce more than one possible solution. Problems with more than one solution will be introduced later. When these problems are initially set, children should revert to using words rather than notation to answer the problem, e.g. 2 and 2-fifths before $2\frac{2}{5}$.
Anticipated response(s)	
	

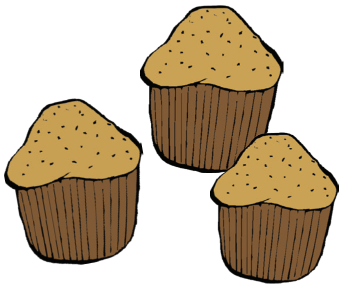
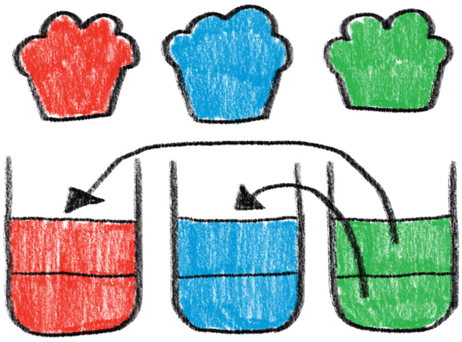
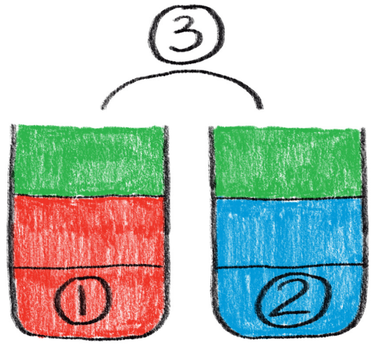
9. Activities that involve re-combining and counting in fractions

NumberSense Workbook problem(s)	Comments
 How many thirds are there in 2? How many thirds are there in 5?	This activity encourages children to notice that three lots of 1-thirds are 3-thirds which is 1 whole. Note how children are starting to use fractions independent of a context that gives meaning to the fraction. Note also how the second part of the question requires children to apply what they have noticed so far. Namely that: 3 lots of $\frac{1}{3}$ is 1 6 lots of $\frac{1}{3}$ is 2 9 lots of $\frac{1}{3}$ is 3 so 5 × 3 = 15 lots of $\frac{1}{3}$

10. Problems that involve developing an awareness that the same fractional amount can be named/described in different ways (equivalent fractions)

NumberSense Workbook problem(s)	Comments
Three children share 5 chocolate bars equally. Show how they must do it. Four children share 6 chocolate bars equally. Show how they must do it. 	The first problem has only one possible solution. The second problem has two possible solutions depending on how the learners share the remaining two chocolate bars. This is because the number of chocolate bars left over is a factor of the number of children sharing the chocolate bars. The role of the teacher is to encourage children to reflect on the two solutions and encourage them to recognise that both fractions ($\frac{1}{2}$ and $\frac{2}{4}$) describe the same amount in different ways. This creates an awareness of equivalent fractions, although not formally introduced yet.
Anticipated response(s) to the second problem	
	

11. Problems that reveal what it means to calculate (divide/multiply) with fractions

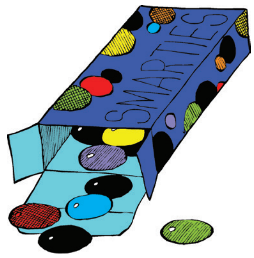
NumberSense Workbook problem(s)	**Comments**
Mrs Singh uses $\frac{2}{3}$ of a cup of flour to make 1 muffin. She wants to make 3 muffins. How many cups of flour does she need? Mrs Singh uses $\frac{2}{3}$ of a cup of flour to make 1 muffin. She has 2 cups of flour. How many muffins can she make? 	These problems provoke children to, in effect, multiply and divide with fractions. Other problems used before these will involve situations that provoke addition and subtraction with fractions. Children should use drawings to make sense of problems such as these. If children are encouraged to solve the problems using drawings, then the problems are well within reach of children at this stage of the journey. Solving the problem in this way supports the development of conceptual understanding. If, however, the teacher expects children to write number sentences (numerical expressions) such as $3 \times \frac{2}{3}$ or $2 \div \frac{2}{3}$ to solve the problems, then these problems are out of reach at this stage.
Anticipated response(s)	
	

12. Tasks that draw attention to the notion of equivalent fractions (different fractions describing the same amount)

NumberSense Workbook problem(s) and anticipated response(s)

There are 12 Smarties in a box.

a. Complete the table.



Fraction of the box	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$
Number of Smarties	1	2	3	4	6

b. Use the table to determine how many Smarties there are in each fraction of the box.

Fraction of the box	Number of Smarties	Fraction of the box	Number of Smarties	Fraction of the box	Number of Smarties
$\frac{2}{12}$	2	$\frac{7}{12}$	7	$\frac{4}{6}$	8
$\frac{3}{12}$	3	$\frac{8}{12}$	8	$\frac{5}{6}$	10
$\frac{4}{12}$	4	$\frac{9}{12}$	9	$\frac{2}{4}$	6
$\frac{5}{12}$	5	$\frac{2}{6}$	4	$\frac{3}{4}$	9
$\frac{6}{12}$	6	$\frac{3}{6}$	6	$\frac{2}{3}$	8

c. Write down all the examples of different fractions that represent the same number of Smarties.

$\frac{4}{12}$, $\frac{2}{6}$ *and* $\frac{1}{3}$ *all represent 4 Smarties*

$\frac{8}{12}$, $\frac{4}{6}$ *and* $\frac{2}{3}$ *all represent 8 Smarties*

$\frac{6}{12}$, $\frac{3}{6}$, $\frac{2}{4}$ *and* $\frac{1}{2}$ *all represent 6 Smarties*

$\frac{9}{12}$ *and* $\frac{3}{4}$ *all represent 9 Smarties*

Comments

These tasks are used to draw attention to the fact that different fractions can be used to describe the same amount, in this case Smarties in a box. The number of Smarties in the box is carefully selected to ensure that there are several different fractions that can be used to describe the same number of Smarties.

Note, it is not expected that the label "equivalent fractions" is introduced or used at this stage.

As the teacher reflects on the task with the class, she may ask children what they notice about the fractions that describe the same amount. Children should notice that in most cases the numerator and denominator have a common factor. Teachers should avoid talking about "multiplying the top and bottom by the same amount".

13. Tasks that rely on the application of observations made in the previous tasks

NumberSense Workbook problem(s) and anticipated response(s)
Complete. a. $\frac{1}{2} = \frac{2}{4}$, $\frac{1}{2} = \frac{3}{6}$, $\frac{1}{2} = \frac{4}{8}$ b. $\frac{1}{4} = \frac{2}{8}$, $\frac{1}{4} = \frac{3}{12}$, $\frac{1}{4} = \frac{4}{16}$ c. $\frac{3}{4} = \frac{6}{8}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{3}{4} = \frac{12}{16}$ d. $\frac{1}{8} = \frac{2}{16}$, $\frac{2}{8} = \frac{4}{16}$, $\frac{3}{8} = \frac{6}{16}$
Comments
These tasks rely on children applying the patterns that they noticed between the numerators and denominators of equivalent fractions. Namely that if the denominator of one fraction is a certain multiple of the denominator of the other fraction, then the numerator is also the same multiple of the numerator of the other fraction. Note, teachers should continue to avoid the language of "multiplying the top and the bottom by the same amount". Doing this introduces a "method" (trick) that cannot be explained/justified. Rather, if children do get stuck, revert to thinking about these pairs of fractions in terms of a number of Smarties in a box. Of course, children may use "multiplying tops and bottoms" in their descriptions of what they notice, teachers should avoid this becoming the "method".

14. Tasks that involve adding and subtracting unlike fractions using a context (a box of Smarties) that is provided

NumberSense Workbook problem(s) and anticipated response(s)
There are 12 Smarties in a box. a. How many Smarties are there in $\frac{1}{6}$ of the box plus $\frac{1}{4}$ of the box? $\frac{1}{6}$ of the box = 2 Smarties $\frac{1}{4}$ of the box = 3 Smarties   2 + 3 = 5 Smarties b. Can you write the number of Smarties as one fraction of the box? 5 Smarties = $\frac{5}{12}$ of the box c. Write $\frac{1}{6} + \frac{1}{4}$ as one fraction. $\frac{1}{6} + \frac{1}{4} = \frac{5}{12}$
Use the thinking you used in the previous question to write each expression as one fraction. a. $\frac{1}{2}^{6} + \frac{1}{3}^{4} = \frac{10}{12} = \frac{5}{6}$ b. $\frac{1}{3}^{4} + \frac{1}{4}^{3} = \frac{7}{12}$ c. $\frac{1}{3}^{4} + \frac{1}{6}^{2} = \frac{6}{12} = \frac{1}{2}$ d. $\frac{1}{2}^{6} + \frac{1}{6}^{2} = \frac{8}{12} = \frac{2}{3}$ e. $\frac{1}{12}^{1} + \frac{1}{3}^{4} = \frac{5}{12}$ f. $\frac{5}{12}^{5} + \frac{1}{4}^{3} = \frac{8}{12} = \frac{2}{3}$ g. $\frac{1}{2}^{6} + \frac{1}{6}^{2} = \frac{8}{12} = \frac{2}{3}$ h. $\frac{2}{3}^{8} + \frac{2}{6}^{4} = \frac{12}{12} = 1$ i. $\frac{2}{3}^{8} + \frac{1}{4}^{3} = \frac{11}{12}$

Comments

In these tasks we want children to develop confidence in adding unlike fractions without explicitly rewriting the unlike fractions as like fractions. Rather, we want, at this stage, that they continue to use the part of a collection model of a fraction to make sense of the question. What is important is that they see the link between this task and their ability to make like fractions from the previous problems. The teacher's important role in facilitating reflection of the task cannot be understated.

15. Tasks that involve adding and subtracting unlike fractions in which the child must choose a common denominator

NumberSense Workbook problem(s) and anticipated response(s)

Calculate $\frac{2}{3} + \frac{1}{5}$.

Oratile

I cannot write thirds as fifths or fifths as thirds. I think about a box of Smarties. I imagine a box of Smarties with 30 Smarties in the box.
$\frac{2}{3}$ of a box of 30 Smarties = 20 Smarties.
$\frac{1}{5}$ of a box of 30 Smarties = 6 Smarties.
So $\frac{2}{3}$ of the box + $\frac{1}{5}$ of the box = 26 Smarties
= $\frac{26}{30}$ of the box.
And so, $\frac{2}{3} + \frac{1}{5} = \frac{26}{30}$.

I also imagine a box of Smarties. But I imagine a box with 15 Smarties in the box.
$\frac{2}{3}$ of a box of 15 Smarties = 10 Smarties.
$\frac{1}{5}$ of a box of 15 Smarties = 3 Smarties.
So $\frac{2}{3}$ of the box + $\frac{1}{5}$ of the box = 13 Smarties
= $\frac{13}{15}$ of the box.
And so, $\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$.

Palesa

Oratile and Palesa think carefully about how many Smarties there should be in a box to help them add the fractions. By choosing the number of Smarties in the box carefully, Palesa ends with a simpler fraction.

Use a "think about a Smartie box" strategy to calculate. Show your thinking.

a. $\frac{1}{2} + \frac{1}{3}$ 6 smarties, then $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

b. $\frac{1}{4} + \frac{1}{6}$ 12 smarties, then $\frac{3}{12} + \frac{2}{12} = \frac{5}{12}$

c. $\frac{1}{6} + \frac{2}{9}$ 18 smarties, then $\frac{1}{6} + \frac{2\times 1}{9} = \frac{3}{18} + \frac{2\times 2}{18} = \frac{7}{18}$

d. $\frac{2}{3} + \frac{3}{4}$ 12 smarties, then $\frac{2\times 1}{3} + \frac{3\times 1}{4} = \frac{2\times 4}{12} + \frac{3\times 3}{12}$
$= \frac{17}{12} = 1\frac{5}{12}$

Discuss how you decide on the number of Smarties you need in each box.

I decide on the smallest number of Smarties that can be divided by both denominators.

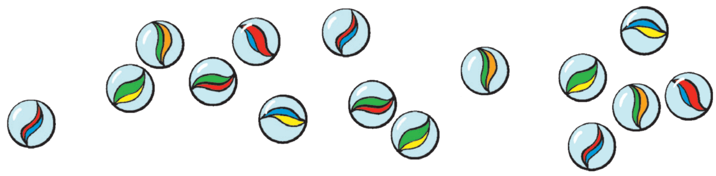
Comments

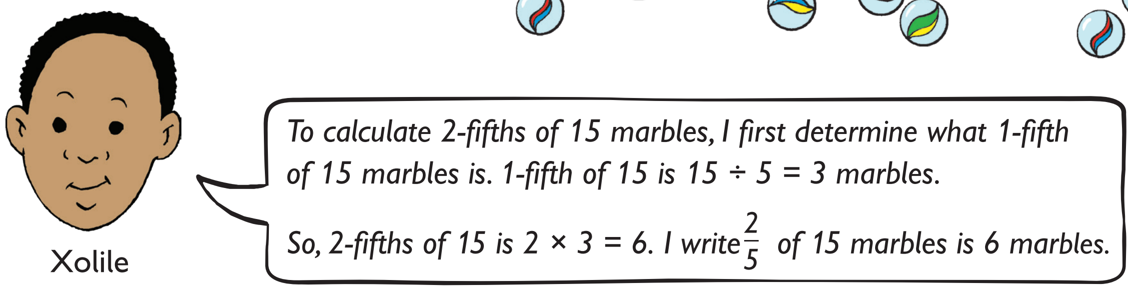
In these tasks we want children to become aware of the relationship between the denominators of the two fractions and the denominator of the like fractions. They do so by choosing an appropriate number of Smarties. Note that this makes greater conceptual sense than teaching "the lowest common multiple of the denominators of the two fractions". Again, the teacher-led reflection session will support children to describe what they notice in words that have the same meaning as "the lowest common multiple of the denominators".

16. Tasks that involve calculating a fraction of a collection

NumberSense Workbook problem(s) and anticipated response(s)

Calculate $\frac{2}{5}$ of 15 marbles.





Xolile

Calculate.

a. $\frac{3}{5}$ of 5 — *1-fifth of 5 = 1; $\frac{3}{5}$ of 5 = 3*

b. $\frac{3}{5}$ of 50 — *1-fifth of 50 = 10; $\frac{3}{5}$ of 50 = 30*

c. $\frac{3}{5}$ of 100 — *1-fifth of 100 = 20; $\frac{3}{5}$ of 100 = 60*

d. $\frac{3}{8}$ of 24 — *1-eighth of 24 = 3; $\frac{3}{8}$ of 24 = 9*

e. $\frac{3}{8}$ of 32 — *1-eighth of 32 = 4; $\frac{3}{8}$ of 32 = 12*

f. $\frac{3}{8}$ of 72 — *1-eighth of 72 = 9; $\frac{3}{8}$ of 72 = 27*

g. $\frac{3}{4}$ of 16 — *1-fourth of 16 = 4; $\frac{3}{4}$ of 16 = 12*

h. $\frac{2}{3}$ of 9 — *1-third of 9 = 3; $\frac{2}{3}$ of 9 = 6*

i. $\frac{5}{6}$ of 12 — *1-sixth of 12 = 2; $\frac{5}{6}$ of 12 = 10*

Comments

To calculate a fraction of a collection is the start of the multiplication of fractions journey. This task relies on children's ability to determine a part of a collection (which they have been doing for many years) and then to determine a multiple of that "part" and in so doing to determine a non-unitary fraction of a collection. At this stage, the collection is deliberately a multiple of the fraction type (fourths, fifths, sixths, eighths, etc).

17. Tasks that involve calculating a fraction of a fraction

NumberSense Workbook problem(s) and anticipated response(s)

Use your experience of calculating fractions of collections to calculate.

a. $\frac{3}{5}$ of 5-eighths

$\frac{1}{5}$ of 5-eighths = 1-eighth

$\frac{3}{5}$ of 5-eighths = 3-eighths

b. $\frac{3}{5}$ of 10-twelfths

$\frac{1}{5}$ of 10-twelfths = 2-twelfths

$\frac{3}{5}$ of 10-twelfths = 6-twelfths

c. $\frac{3}{5}$ of 25-thirtieths

$\frac{1}{5}$ of 25-thirtieths = 5-thirtieths

$\frac{3}{5}$ of 25-thirtieths = 15-thirtieths

d. $\frac{3}{8}$ of 16-twentieths

$\frac{1}{8}$ of 16-twentieths = 2-twentieths

$\frac{3}{8}$ of 16-twentieths = 6-twentieths

e. $\frac{3}{8}$ of 32-thirty-fifths

$\frac{1}{8}$ of 32-thirty-fifths = 4-thirty-fifths

$\frac{3}{8}$ of 32-thirty-fifths = 12-thirty-fifths

f. $\frac{3}{8}$ of 24-thirtieths

$\frac{1}{8}$ of 24-thirtieths = 3-thirtieths

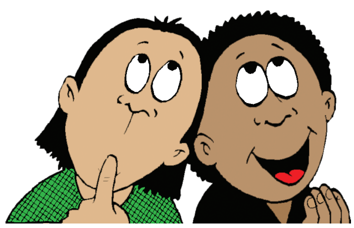
$\frac{3}{8}$ of 24-thirtieths = 9-thirtieths

Comments

To determine a fraction of a fraction is in effect to multiply two fractions together. By writing the second fraction using words this task links naturally to calculating a fraction of a collection.

5-eighths is a collection of 5 one-eighths or $5 \times \frac{1}{8}$ and 16-twentieths is a collection of 16 one-twentieths or $16 \times \frac{1}{20}$

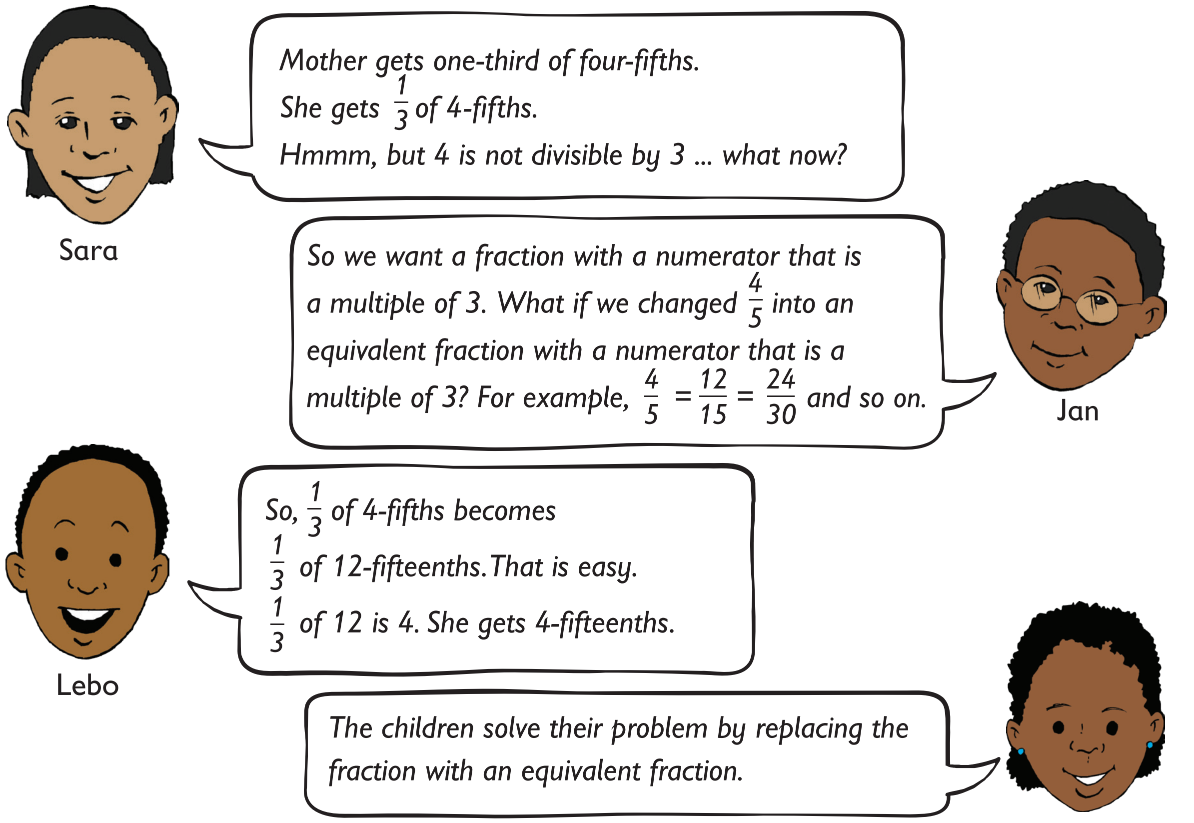
The denominator of the second fraction is a unit, just as kg and m and $ are units (a denomination). This is where the word denominator comes from. In these tasks the number in the collection (the numerator) is deliberately a multiple of the first fraction to make calculating the fraction of the fraction collection easier.



18. Tasks that involve calculating a fraction of a fraction where the number in the collection (the numerator) of the second fraction is not a multiple of the fraction.

NumberSense Workbook problem(s) and anticipated response(s)

Mother buys a gatsby. She gives one-fifth of the gatsby to father and shares the rest of the gatsby between herself and her two children. How much does mother get?



Use a "replace the fraction with an equivalent fraction" strategy to calculate.

a. $\frac{1}{3}$ of 5-sixths

5-sixths = 15-eighteenths

$\frac{1}{3}$ of 15-eighteenths = 5-eighteenths

$= \frac{5}{18}$

b. $\frac{1}{4}$ of 3-fifths

3-fifths = 12-twentieths

$\frac{1}{4}$ of 12-twentieths = 3-twentieths

$= \frac{3}{20}$

c. $\frac{1}{5}$ of 2-thirds

2-thirds = 10-fifteenths

$\frac{1}{5}$ of 10-fifteenths = 2-fifteenths

$= \frac{2}{15}$

d. $\frac{1}{3}$ of $\frac{4}{5}$

4-fifths = 12-fifteenths

$\frac{1}{3}$ of 12-fifteenths = 4-fifteenths

$= \frac{4}{15}$

e. $\frac{1}{4}$ of $\frac{5}{6}$

5-sixths = 20-twenty-fourths

$\frac{1}{4}$ of 20-twenty-fourths = 5-twenty-fourths

$= \frac{5}{24}$

f. $\frac{1}{5}$ of $\frac{3}{4}$

3-fourths = 15-twentieths

$\frac{1}{5}$ of 15-twentieths = 3-twentieths

$= \frac{3}{20}$

g. $\frac{2}{3}$ of $\frac{2}{3}$

2-thirds = 6-ninths

$\frac{2}{3}$ of 6-ninths = 4-ninths

$= \frac{4}{9}$

h. $\frac{3}{4}$ of $\frac{7}{10}$

7-tenths = 28-fourtieths

$\frac{3}{4}$ of 28-fourtieths = 21-fourtieths

$= \frac{21}{40}$

i. $\frac{4}{5}$ of $\frac{3}{7}$

3-sevenths = 15-thirty-fifths

$\frac{4}{5}$ of 15-thirty-fifths = 12-thirty-fifths

$= \frac{12}{35}$

Comments

In these tasks the questions looks more like multiplying two fractions together. However, to develop deep conceptual understanding of what it means to multiply fractions, it is important to continue to think of the second fraction as a collection of units (unit fractions). The problem is that the number of unit fractions in the collection is not a multiple of the denominator of the first fraction. To solve the problem we exchange the second fraction with an equivalent fraction in which the collection of unit fractions is a multiple of the denominator of the first fraction. Converting one fraction for another is no different from converting between currencies $ to € or converting between grams and kilograms, etc.

If, in the teacher-led reflection, children notice a pattern in the solution, namely that it looks like they are "multiplying the tops" and "multiplying the bottoms" then so be it. BUT teachers are encouraged not to teach this method (it does not develop understanding). The point is that while it may look like "multiplying the top" and "multipying the bottom" that is not what is actually happening.

Teaching problem-solving strategies

Problem solving is knowing what to do when you don't know what to do.

To be effective problem-solvers, teachers and children need to develop both a range of problem-solving strategies (heuristics) as well as an understanding of the problem-solving process.

Problem-solving process

George Pólya, a Hungarian mathematician, described a four-step process for problem solving in his book "How to Solve It" first published in 1944. These steps are:

- Understand the problem: Clarify what is being asked.
- Devise a plan: Select a strategy to tackle the problem.
- Carry out the plan: Implement the strategy.
- Look back: Review the solution to see if it can be improved or if there is another way to solve the problem.

These steps are designed to provide a systematic approach to solving problems and to develop critical thinking.

George Pólya also stated that "If there is a problem you can't solve, then there is an easier problem you can solve: find it" which speaks to the need for children to learn how to break complex problems into simpler, more manageable problems.

It is important that teachers remind children of these steps when they solve problems.

Problem-solving strategies

In the same way that a mechanic has a range of different tools in their toolbox that they select from depending on the task that they are busy with, to solve problems, teachers and children need to have a range of problem-solving strategies at their disposal.

Act out the problem

The "act out the problem" strategy involves using counters, toys, or even people to physically act out the problem. Acting out the problem can also help the problem-solver to better understand the problem before solving it.

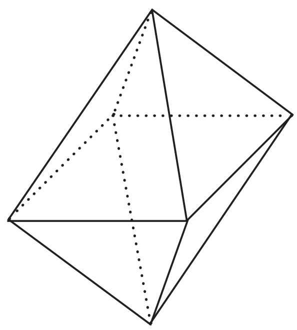
Acting out the problem is particularly useful with young learners who are not ready to draw pictures of a situation and/or to write their solution.

Example 1:

If there are 4 people in a room and each person shakes hands with every other person exactly once, how many handshakes occur?

Example 2:

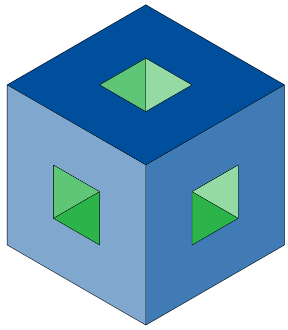
The faces of a regular octahedron are painted so that any two faces with an edge in common have different colours. What is the minimum number of different colours needed?



Australian Mathematics Competition Book 3 1992-1998 p.85 q.1

Example 3:

A solid cube with a side length of 3 metres has square holes cut through the centre of each face, reaching the centre of the opposite face. The holes intersect in the middle of the cube, forming square openings with sides of 1 metre. What is the total surface area, in square metres, of the modified cube?

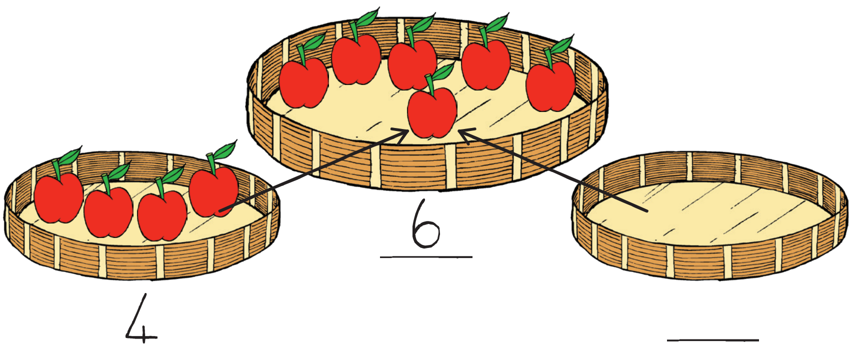


Australian Mathematics Competition Book 3 1992-1998 p.79 q.4

For the example that has been allocated to your group, solve the problem and describe how you used the "act out the problem" strategy to do so.

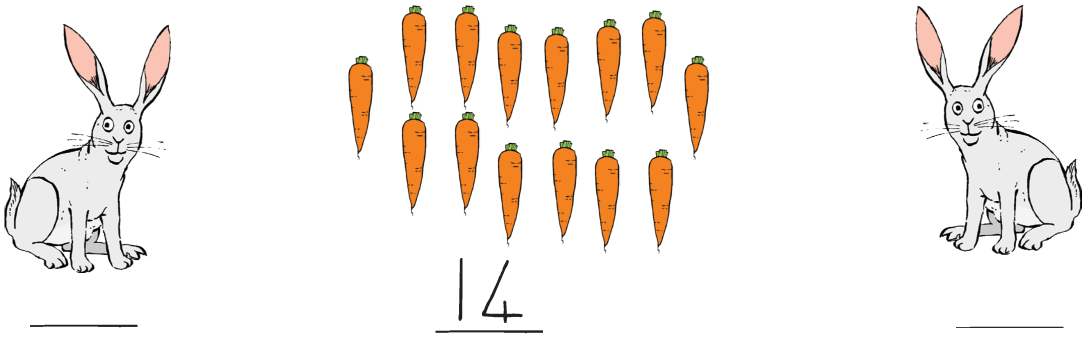
NumberSense Workbook examples:

Complete the picture. Write the numbers.



NumberSense Workbook 0 p. 43

Share the carrots equally. Write the numbers.



NumberSense Workbook 0 p. 44

Draw a picture

The "draw a picture" strategy involves representing the problem by means of a drawing to better understand it. Drawing a picture can help to clarify the elements of the problem as well as the relationship(s) between them. A picture can also help to explain the problem and its solution to others. Using the "draw a picture" strategy is not limited to young children, professional mathematicians also use pictures to better understand problems and to illustrate their solutions.

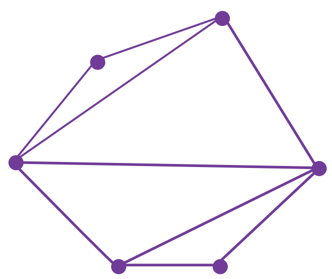
Example 1:

What is the smallest number of children a family must have so that each child has at least one brother and at least one sister?

Australian Mathematics Competition Book 3 1992-1998 p.92 q.2

Example 2:

We want to place six points on a plane and connect them with straight lines, ensuring no lines intersect. The diagram shows 9 lines, but by rearranging the points, more lines can be drawn. If you can place the points anywhere, what is the maximum number of lines you can draw?



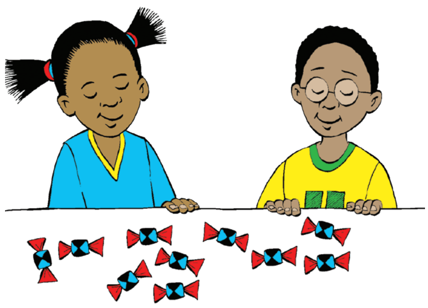
Australian Mathematics Competition Book 3 1992-1998 p.104 q.12

For the example that has been allocated to your group, solve the problem and describe how you used the "draw a picture" strategy to do so.

NumberSense Workbook examples:

Jan and Fundi share 10 toffees equally.

Show them how to do it.



NumberSense Workbook 3 p. 31

Ben has 18 marbles. He puts 4 marbles in a bag.

How many bags can he fill?

NumberSense Workbook 4 p. 17

Look for a pattern

The "look for a pattern" strategy, is often used in conjunction with the "make a list" or "use a table" strategies. The strategy involves exploring different cases or a range of possible solutions to the problem, looking for patterns in the data and, based on the pattern, predicting what comes next in the sequence which in turn leads to solving the problem.

Example 1:

How many positive integers less than 900 are multiples of 7 and end in 2?

Australian Mathematics Competition Book 3 1992-1998 p.93 q.8

Solve the problem and describe how you used a "look for a pattern" strategy to do so.

NumberSense Workbook examples:

Complete this row of numbers.

5 ; 10 ; 15 ; 20 ; ___ ; ___ ; ___ ; ___

- What will the fifth number in the row be? ____
- What will the tenth number in the row be? ____
- What will the twelfth number in the row be? ____
- What will the twentieth number in the row be? ____

So what is:

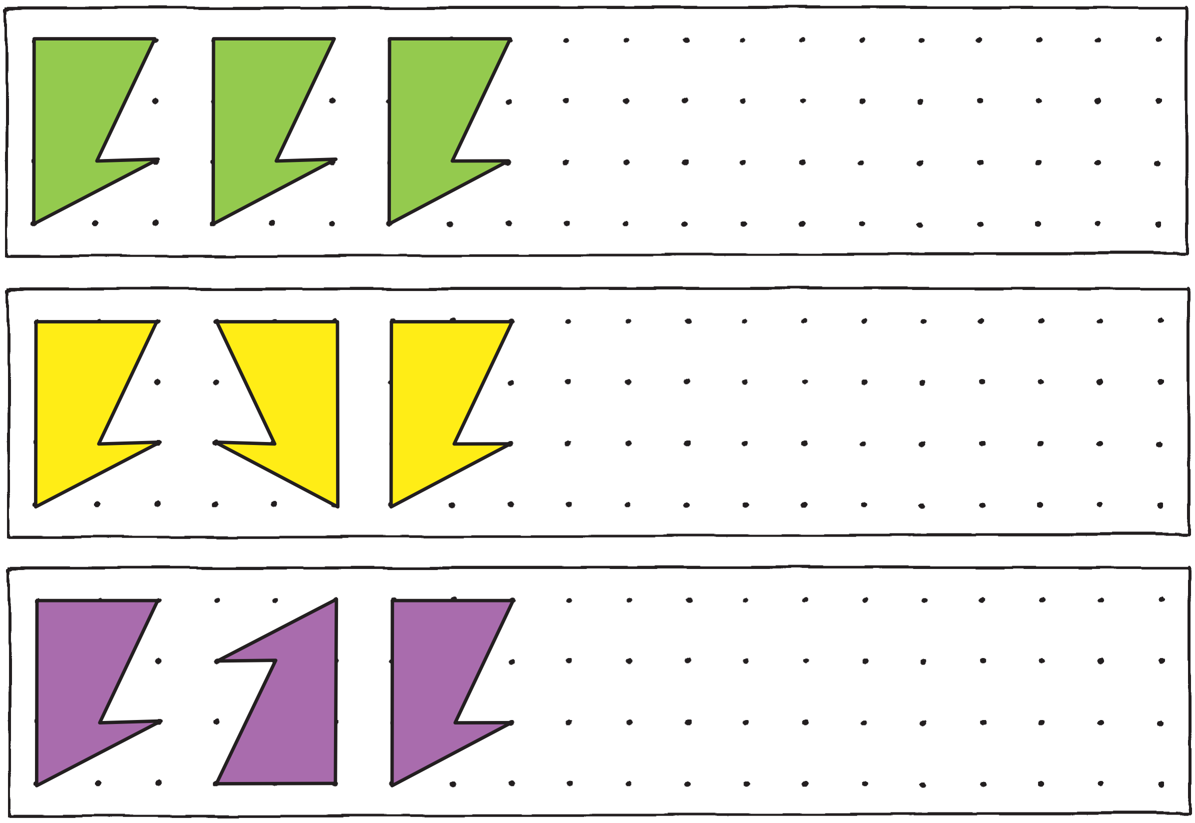
- $5 \times 5 =$ ____
- $10 \times 5 =$ ____
- $12 \times 5 =$ ____
- $20 \times 5 =$ ____

NumberSense Workbook 9 p. 19

Trace the shape onto a piece of card and cut it out.

Use the shape to extend the patterns.

Below each pattern describe what you did to extend the pattern.



NumberSense Workbook 17 p. 39

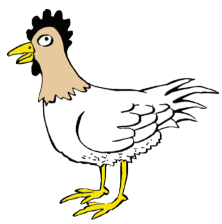
Guess and check

The "guess and check" strategy is useful if there are a limited number of solutions, and when another approach is not apparent. The strategy involves making a guess (preferably an educated guess), checking to see if the solution solves the problems and if not, changing/improving the guess until the problem is solved.

Example 1:

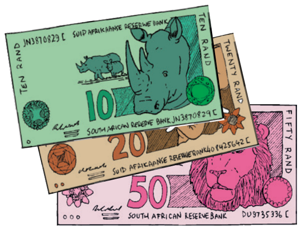
Mr Jones has a total of 25 chickens and cows on his farm.

How many of each does he have if altogether there are 76 feet?



Example 2:

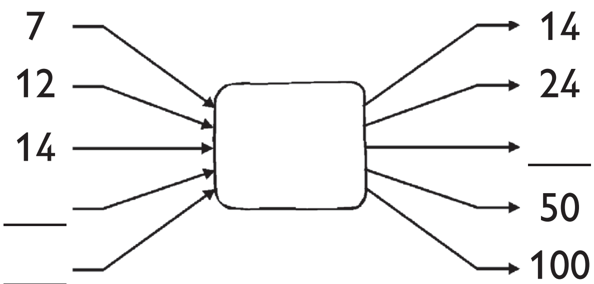
Tina has twice as much money as Jabu. If Tina spends R120 and Jabu spends R48 they both have the same amount of money left over. How much money did Tina have in the beginning?

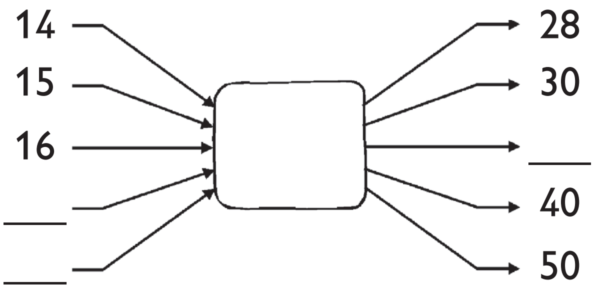


Show how the "guess and check" strategy can be used to solve the problem.

NumberSense Workbook examples:

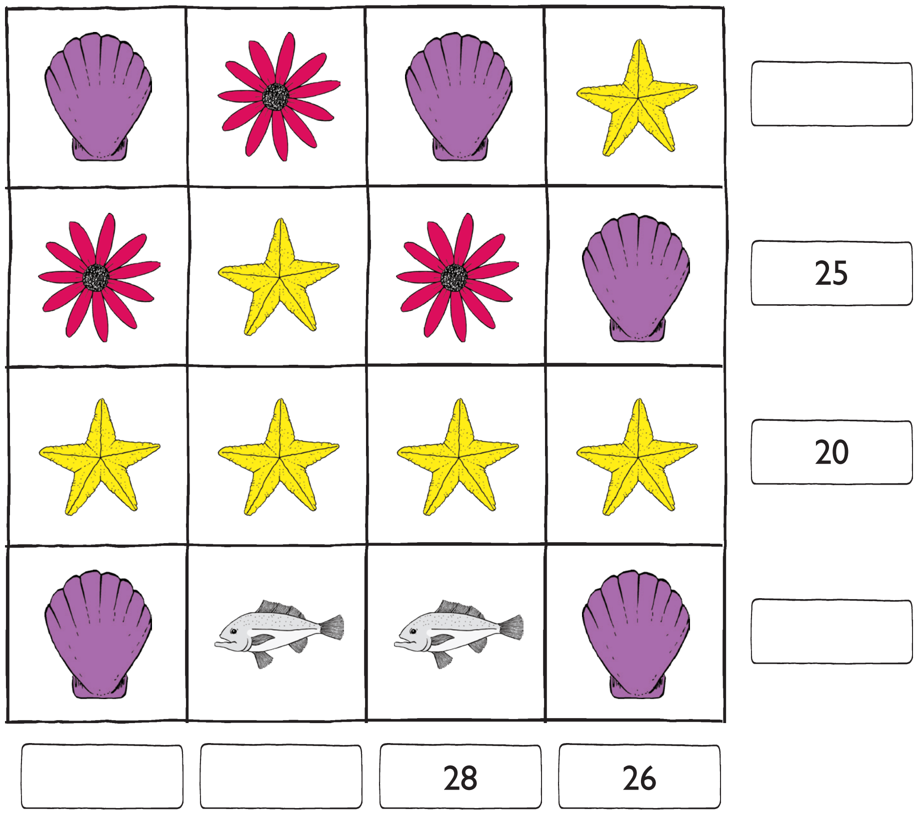
Complete.





NumberSense Workbook 9 p. 10

Each shape represents a different number. If you add up the numbers, you will get the totals shown. Calculate the other totals.



NumberSense Workbook 18 p. 39

Work backwards

The "work backwards" strategy involves starting with the end in mind and thinking about the steps that are needed to reach the outcome. Solving equations is a good example of working backwards from the output to the input value. In the context of problem solving, knowing what the outcome is and working backwards to think about what is needed to reach the outcome can help to solve the problem.

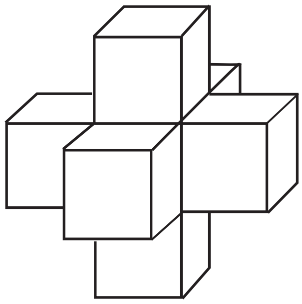
Example 1:

Thato's mother left a plate of cookies on the counter. By the end of the day, there were 5 cookies left. How many cookies were there to start with if Thato ate 2, father ate 3, and mother gave 12 to the neighbour?

Australian Mathematics Competition Book 3 1992-1998 p.78 q.2

Example 2:

Seven cubes are glued together face to face as shown. If the total volume is 448 cubic centimetres, what is the surface area, in square centimetres, of the solid?



Australian Mathematics Competition Book 3 1992-1998 p.78 q.2

For the example that has been allocated to your group, solve the problem and describe how you used the "work backwards" strategy to do so.

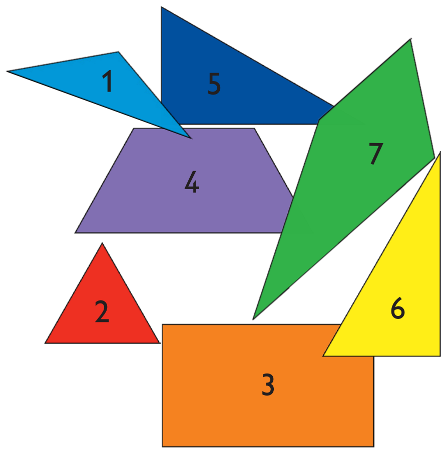
NumberSense Workbook examples:

Show that:

Piece 5 covers the same amount of space as piece 1 and piece ______.

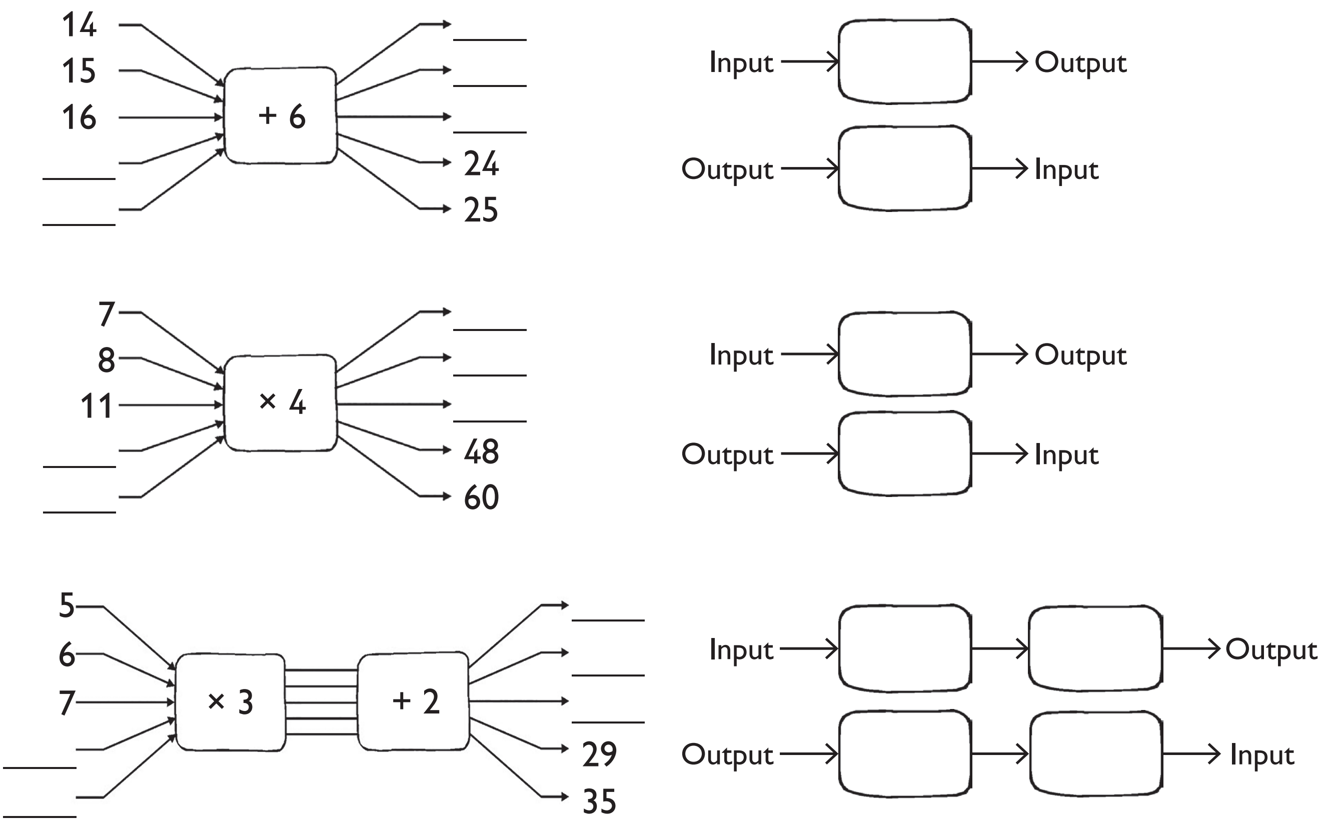
Piece 3 covers the same amount of space as piece 5 and piece ______.

Piece 4 covers the same amount of space as piece ______ and piece ______.



NumberSense Workbook 12 p. 57

Complete.



NumberSense Workbook 24 p. 5

Make a list

The "make a list" strategy involves organising information into a list format to systematically explore all possible solutions. It's particularly useful when dealing with problems that have multiple solutions or require a combination of elements. Using the strategy involves writing down all the possible options in a logical order, going through the list and evaluating which of the items in the list solve the problem. The strategy involves using a systematic approach to solving a problem.

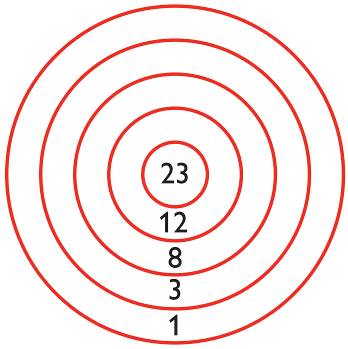
Example 1:

How many even four-digit numbers can be formed using the digits 1, 2, 3, and 5?

Australian Mathematics Competition Book 3 1992-1998 p.93 q.3

Example 2:

Three darts are thrown at a dartboard. The scores from each throw are added together, with a miss counting as zero. What is the smallest total score that cannot be achieved?



Australian Mathematics Competition Book 3 1992-1998 p.22 q.19

For the example that has been allocated to your group, solve the problem and describe how you used the "make a list" strategy to do so.

NumberSense Workbook example:

Show in how many different ways you can make:

- 60c
- 75c
- 150c

NumberSense Workbook 6 p. 43

Mrs Sibusa plants 72 mealie plants in rows.

How many different ways can she plant them?

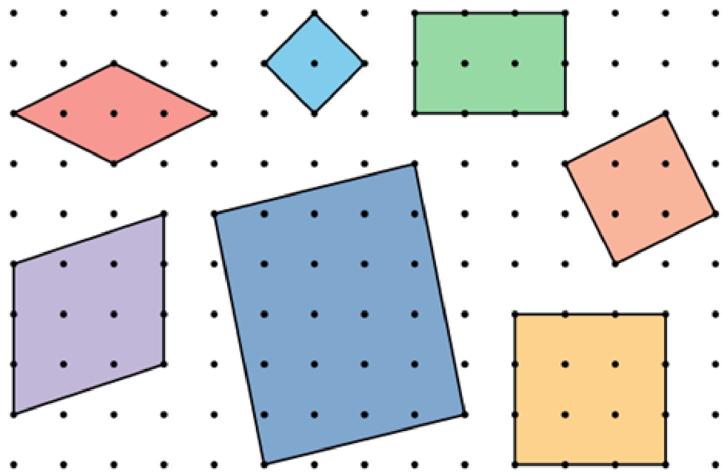
NumberSense Workbook 6 p. 17

Explore special cases

The "explore special cases" strategy involves looking for specific (special) cases to better understand the problem. This strategy can help to identify patterns, properties, exceptions, and unique conditions that may not be obvious when first looking at the problem. This strategy is very helpful when determining general properties or solutions.

Example 1:

Which of the quadrilaterals are squares? Select all that apply.



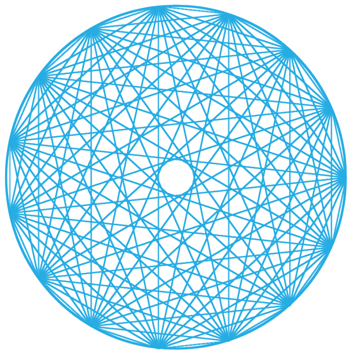
Solve the problem and describe how you used the "explore special cases" strategy to do so.

Solve a simpler problem

The "solve a simpler problem" strategy is what Polya had in mind when he said: "If there is a problem you can't solve, then there is an easier problem you can solve: find it and solve it." The "solve a simpler problem" strategy is a powerful strategy which involves breaking a complex problem into smaller more manageable parts, solving those, and then looking for a pattern that will help solve the more complex problem.

Example 1:

In this figure there are 15 points on the circle, and every point is connected to every other point on the circle. How many connecting lines are there altogether?



SAMC Third Round Junior Paper 2021

Solve the problem and describe how you used the how you can use a "solve a simpler problem" strategy to do so.

NumberSense Workbook example:

Thembi has 3 pairs of shoes: a black pair, a white pair and a brown pair.
She has 2 skirts: a blue one and a white one.
She has 3 T-shirts: a pink one, a green one and a purple one.
How many different outfits can she make with her clothes?
Show your working.

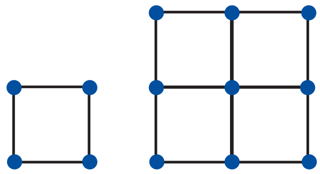
NumberSense Workbook 23 p. 38

Make a table

The "make a table" strategy is similar to the look for a pattern and "make a list" strategies. Making a table is useful for organising data and looking for a pattern which can be used to solve the problem.

Example 1:

A 1 × 1 square is made using 4 matches and a 2 × 2 square, with all the unit squares inside, uses 12 matches as shown. How many matches are needed to construct a 20 × 20 square, including all the unit squares inside?



Australian Mathematics Competition Book 3 1992-1998 p.95 q.15

Solve the problem and describe how you used the how you can use a "make a table" strategy to do so.

NumberSense Workbook examples:

There are 15 players in a rugby team. Complete the table.

Rugby teams	1	2	3	5	10	20	21	30
Players	15							

What is?

15 × 20 = ___

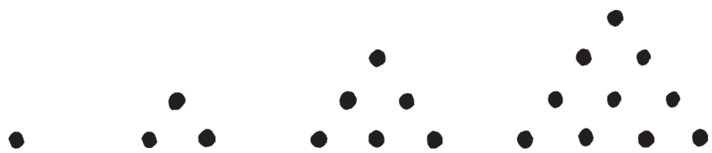
45 ÷ 15 = ___

15 × 22 = ___

90 ÷ 15 = ___

NumberSense Workbook 12 p. 28

Bongi makes pictures with dots like this. The first 4 pictures make a pattern.



Picture 1 Picture 2 Picture 3 Picture 4 Picture 5 Picture 6

a. Draw the fifth and sixth pictures in the pattern.

b. Complete the table.

Picture number	1	2	3	4	5	6	7	8	9	10
Number of dots	1	3	6	10						

c. How many dots will she need for picture 12?

NumberSense Workbook 13 p. 26

Write an equation

The "write an equation" strategy involves developing a formula or equation to represent a problem or situation by means of a mathematical equation. Having developed an equation to represent the situation enables the use of algebra to determine the solution(s) to the problem.

Example 1:

What is the maximum number of Mondays that can occur within a 45-day period?

Australian Mathematics Competition Book 3 1992-1998 p.92 q.1

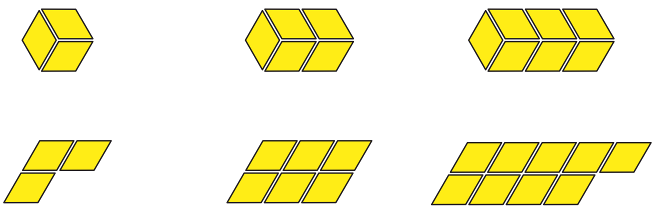
Solve the problem and describe how you used the how you can use a "write an equation" strategy to do so.

NumberSense Workbook examples:

Alan and Liam make pictures with tiles like this . The first three pictures make a pattern. The patterns start with three tiles in picture 1.

Alan

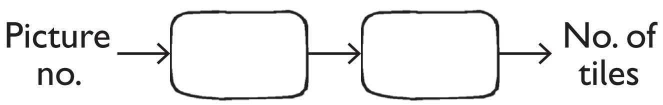
Liam



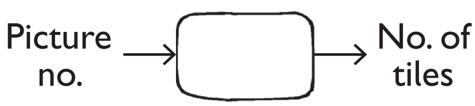
Picture 1 Picture 2 Picture 3 Picture 4 Picture 5

a. Draw the fourth and fifth pictures in each pattern.

b. Select and complete the appropriate flow diagram for the number of tiles in Alan and Liam's patterns. Explain.



________'s pattern.



________'s pattern.

NumberSense Workbook 19 p. 32

Investigations

Children who can solve mathematical problems - in particular unfamiliar problems - are not only more likely to succeed in mathematics, but are also better prepared for the demands of the world after school. In the world of work, and more generally, people need to be effective decision makers and problem-solvers. They must be able to analyse problems, to develop solution strategies, to collect, organise and interpret data, to reason logically, and to communicate their ideas and findings to others. Investigations provide a good context for developing these skills.

Investigations create opportunities for learning and for children to apply their existing mathematical knowledge in new and unfamiliar situations. This in turn develops both a deeper understanding of their existing mathematical concepts and knowledge, and supports the development of new concepts and knowledge.

Investigations vs. problem solving

Investigating and problem solving are in many ways quite similar and people often use the terms to mean the same thing. Both refer to situations that are problematic in nature and which require the application of mathematical knowledge. However, there are some important differences.

In the case of problems, the conditions and the question are both well-defined and while the solution strategy may not be clear, problems are typically well stated and mostly only have one solution.

By contrast, a mathematical investigation typically stresses mathematical processes such as searching for patterns, formulating, testing, justifying and proving conjectures, reflecting, and generalising. With investigations, the question is not always clear and making it clear - problem posing - is the first step in an investigation.

Mathematical investigations present situations that are interesting and capture the attention of the child.

Why use investigations?

Why use investigations?

- Investigations provide opportunities for children to apply their mathematical knowledge, concepts, skills, and strategies in unfamiliar settings.

- In addition to strengthening and reinforcing existing mathematical knowledge, concepts, skills, and strategies, investigations can be used to introduce new mathematics.
- Investigations create opportunities for children to apply and develop their problem-solving skills.
- Investigations are typically completed in groups and, as a result they support children in developing their communication and other interpersonal skills. In addition, because of working together to solve a problem, children must learn to evaluate the reasoning of others and to present their own convincing arguments or justifications.
- Investigations mostly take place over an extended period of time (more than one lesson), therefore they provide children with time for thinking and revising strategies etc.

Steps in completing an investigation

Conducting an investigation typically involves the following steps:

- Pose a question(s): A key characteristic of investigations is that the question is not clear; therefore, the first step is for the child to formulate a question or series of questions that they want to answer by conducting the investigation.
- Collect data: Next, the child needs to research the relevant information or to collect the data needed to answer the question. This could involve measuring and observing.
- Explore and investigate, searching for patterns in the data: Having understood the question(s) and collected the data, the next step involves working with the data. This means looking for patterns, relationships, and connections within the data. It is these patterns, relationships and connections that help to answer the question(s).
- Develop answers to the question(s) posed: Based on their exploration and the question(s) posed, children must develop (formulate) answers and/or solutions, and be able to explain their findings clearly.
- Communicate: Children must be able to communicate their processes, findings, and learnings with others. The sharing of the answers to question(s) posed improves the understanding of the solution by the child.
- Evaluate and reflect: The last step of an investigation involves assessing the accuracy of the responses and reflecting on what has been learned through the investigation.

The role of the teacher in investigations

First and foremost, teachers who use investigations in their mathematics classes need to reflect on what it means to do mathematics and how mathematics is learnt. Investigations are messy, where developing the solutions does not follow a linear, well-defined process (steps). Therefore, the solutions may vary. Investigations are more about the why than the how.

The role of the teacher as the investigation is conducted, changes over time.
Before the investigation, teachers should:

- Identify/select/develop the investigation(s) to be conducted by the children. A good investigation:
    - Must be engaging and encourage exploration. It should provoke the curiosity of children, encouraging them to ask questions and engage in hands-on activities.
    - Should be open-ended, allowing children to approach the task(s) in different ways Although it is common to say that in an investigation there is no single "correct" solution, and children must explore and produce multiple solutions, this is also naïve. There are better and worse solutions and some investigations are framed more tightly than others, leading to fewer possible variations in the responses.
    - Should get children to use a range of different problem-solving skills and strategies.
    - Should integrate mathematical concepts. An investigation could integrate different areas of mathematics, e.g. geometry, measurement, and data handling, all in one investigation.
- Anticipate resources that children may need. This could include construction or modelling equipment, measuring equipment and so on.
- Anticipate questions that children may ask as well as how to respond to the questions without "doing" the investigation.

During the investigation, teachers should:

- Ensure that children understand the problem(s) that the investigation is attempting to answer.
- Facilitate the investigation by the children. This means supporting the children as they conduct the activity, without telling them what to do.

Both may involve asking prompting questions such as:

- What do we know?
- What do we want to know?
- What is the same?
- What is different?
- Can you see a pattern?
- How can this pattern help you?
- What do think comes next? Why?
- What would happen if ... ?
- How did you find that out?
- Why do you think that?
- Have you thought of another way that this could be done?
- Do you think we have found the best solution?

- Assess the children's performance. This could include:
  - The use of mathematical language, mathematical concepts, and skills.
  - The use of problem-solving strategies.
  - The contribution of individuals to the design of the plan and involvement in executing the plan, as well as their perseverance.
  - Collaboration skills, i.e., how well are the children working with others?

After the investigation, teachers should:

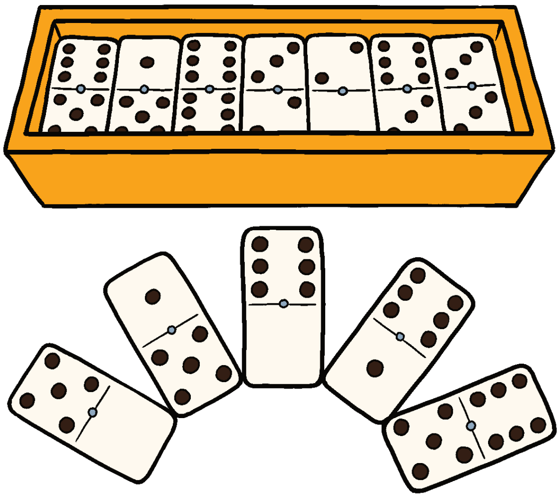
- Evaluate children's written and oral presentations. Ask questions such as:
  - Was the problem-solving strategy the best strategy?
  - Was the sample size appropriate?
  - Did the children use appropriate mathematical concepts, and is the mathematical reasoning sound?

Types of investigations

There are typically three different types of investigations:

Whimsical investigations involve playful or imaginative tasks that encourage creativity in applying mathematical concepts.

DOMINOES



Before you start this investigation, research and determine:

- The different dominoes there are in a standard set of 28 dominoes.

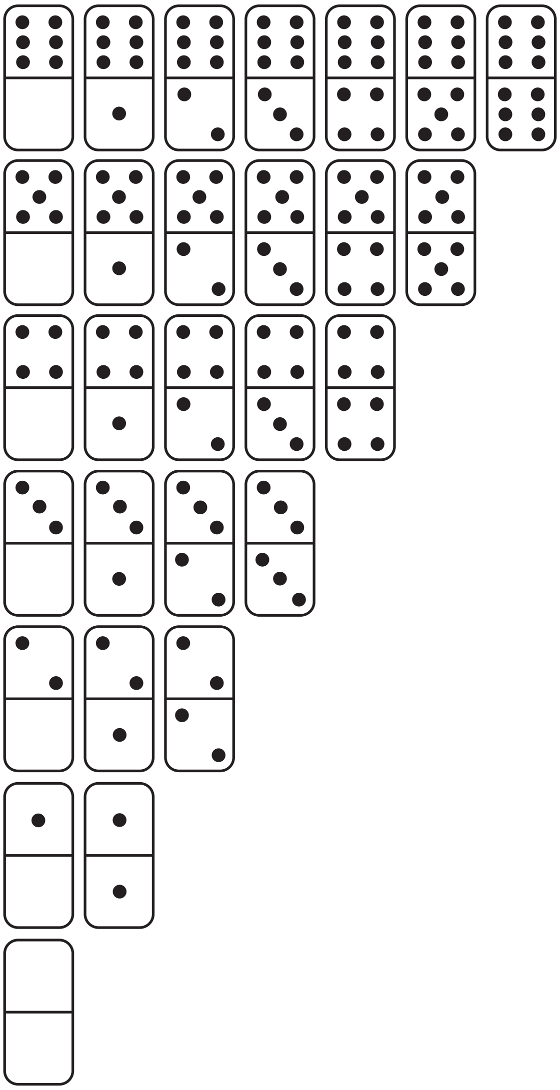
Investigation 1:

Sally has an incomplete set of dominoes. She has 24 dominoes and the total number of spots on the dominoes is 125. Make a list of all the different combinations of dominoes that might be missing. How can you be sure that you have them all?

Investigation 2:

Lerato has an incomplete set of dominoes. She has 24 dominoes and the total number of dots on the dominoes is 160. Make a list of all the different combinations of dominoes that might be missing. How can you be sure that you have them all?

Prepare a report on your findings for submission.



NOTES

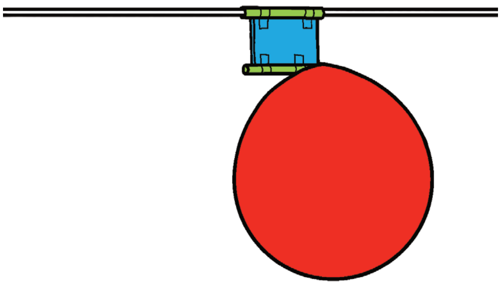
Real investigations focus on the real-world applications of mathematics. Their purpose is to show children how mathematics is used to solve problems in everyday life and can involve data collection and analysis from real events or phenomena.

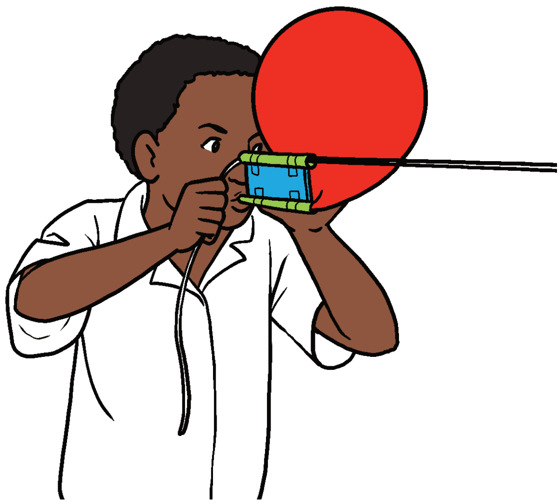
HOW FAR DOES A BALLOON TRAVEL

To complete this investigation, you will need:

- A few drinking straws
- A long piece of thin string that easily goes through the straw (about 6 metres)
- Several balloons
- Two squares of stiff cardboard (6 cm by 6 cm)
- Some cellotape and some cardboard.

Use the materials to make a rocket like the one in the drawing.





<u>Investigation:</u>

Investigate how the number of breaths that you use to blow up the balloon changes the distance that the balloon travels,

For example, if you double the number of breaths, does the balloon travels twice as far?

Prepare a report on your investigation for submission.

NOTES

Mathematical investigations involve one or more area of mathematics. They can be used to introduce children to new mathematical concepts and knowledge. These investigations may include investigations of mathematical puzzles etc.

CONSECUTIVE NUMBERS

3 can be written as the sum of non-zero, consecutive whole numbers in one way:

1 + 2 = 3

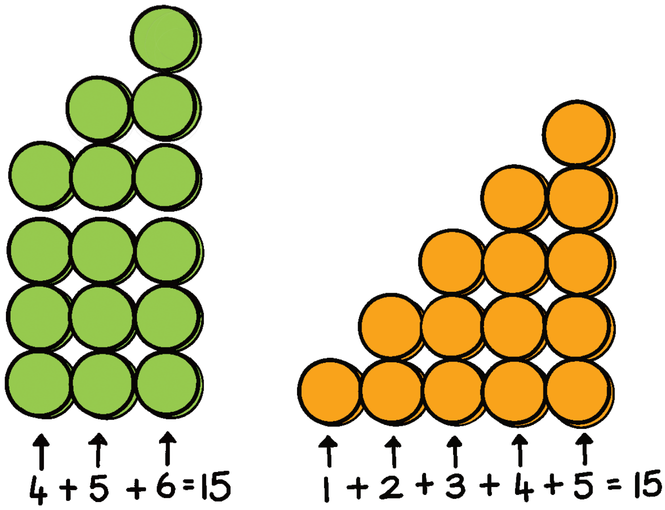
4 cannot be written as the sum of non-zero, consecutive whole numbers.

15 can be written as the sum of non-zero, consecutive whole numbers in three ways:

7 + 8 = 15

4 + 5 + 6 = 15

1 + 2 + 3 + 4 + 5 = 15



<u>Investigate:</u>

- All of the different ways, if any, that the numbers 1 to 50 can be written as the sum of two or more non-zero, consecutive whole numbers.
- If there is a rule to determine all the numbers from 1 to 50 that can be written as the sum of:
  - two non-zero, consecutive whole numbers
  - three non-zero, consecutive whole numbers
  - four non-zero, consecutive whole numbers etc.

Prepare a report on your findings for submission.

NOTES

Multiplication table geometric pattern investigation

(Using Tile Generators in Schools, Association of Teachers of Mathematics (UK), 1994)

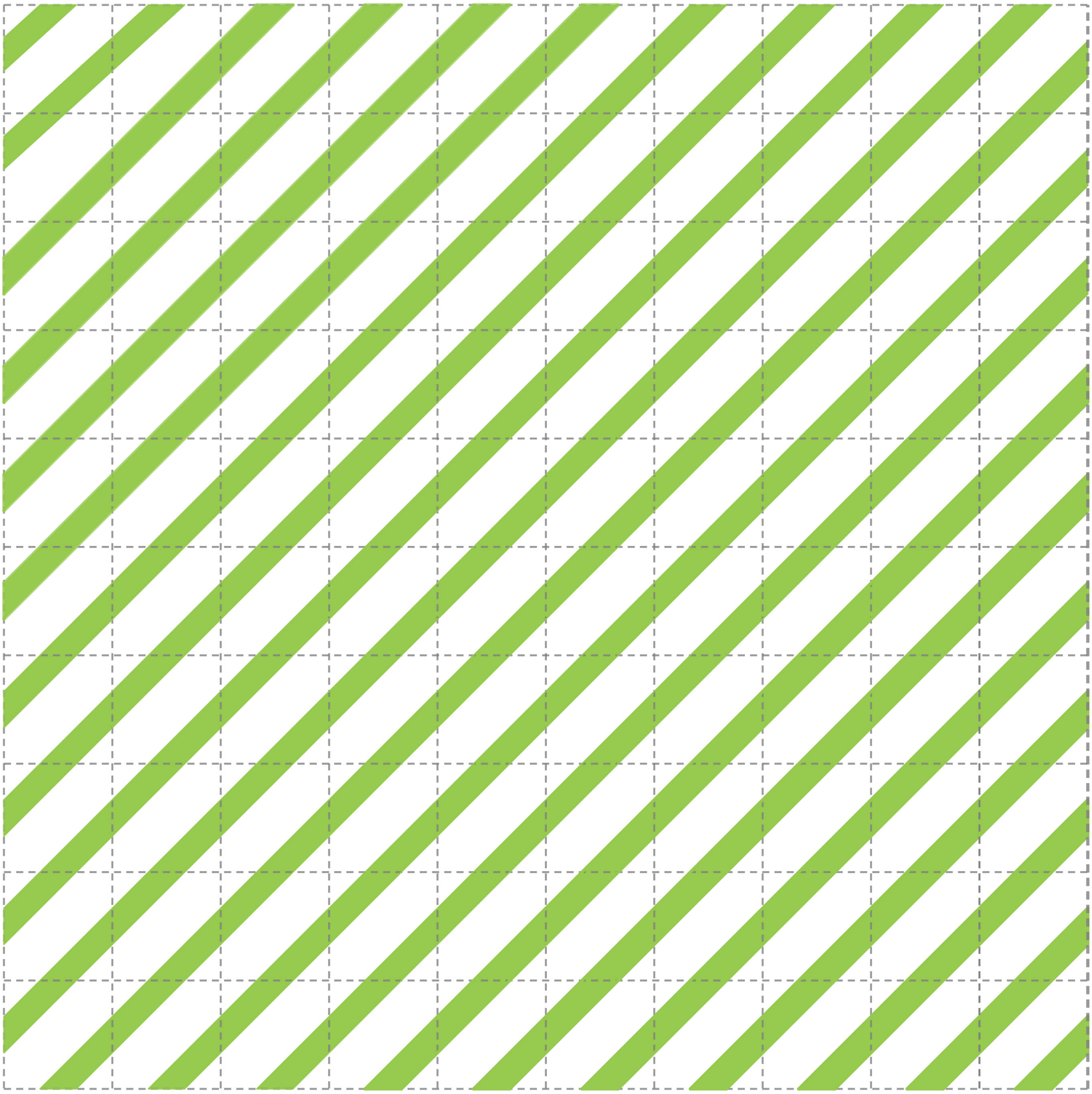
<u>Instructions</u>

- Carefully cut out the tiles on page 61
- For the multiplication table that has been allocated to you, circle all the numbers in the 100 grid. For example, for the 6 times table, circle the numbers 6; 12; 18; 24 and so on.
- Paste the tile on the grid as follows:
  - o For the circled numbers paste the tiles on the grid so that the lines slope to the right.

  - o For the un-circled numbers paste the tiles in the opposite direction.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

Tiles for the geometric pattern investigation



Investigations in the NumberSense workbooks

There are many activities throughout the NumberSense workbooks that are investigations. Teachers are encouraged to treat them in this way.Teachers may also want to treat these as a tasks or investigations for the children's assessment portfolios.

Workbook	**Page**	**Content area**	**Investigation type**
Workbook 2	p. 62/63	Measurement	Real-world
Workbook 6	pp. 60 – 63	Data Handling	Real-world
Workbook 9	pp. 47 – 48	Space & Shape	Mathematical
Workbook 10	p. 49	Space & Shape	Mathematical
Workbook 11	p. 59	Measurement	Real-world
Workbook 12	p. 64	Probability	Real-world
Workbook 13	p. 44/45	Space & Shape	Mathematical
Workbook 14	p. 44	Space & Shape	Mathematical
Workbook 14	p. 63 q2	Measurement	Mathematical
Workbook 15	p. 38/39	Space & Shape	Mathematical
Workbook 15	p. 40	Space & Shape	Mathematical
Workbook 15	pp. 41 – 42	Space & Shape	Mathematical
Workbook 15	pp. 57 – 62	Data Handling	Real-world
Workbook 18	p. 43	Space & Shape	Mathematical
Workbook 18	p. 58/59	Measurement	Mathematical
Workbook 19	p. 22	Number	Whimsical
Workbook 19	p. 38	Number	Whimsical
Workbook 19	pp. 42 – 43	Space & Shape	Whimsical/Mathematical
Workbook 19	pp. 53 – 59	Data Handling	Whimsical
Workbook 23	pp. 56 – 63	Data Handling	Whimsical
Workbook 24	p. 45/46	Space & Shape	Mathematical

NumberSense Website Resources

NumberSense Mathematics Programme Overview

Information that supports the design and implementation of the NumberSense Mathematics Programme.

NumberSense.co.za → About → Programme Overview

NumberSense Workbook Overview

Information and support for the implementation of the NumberSense Mathematics Programme.

NumberSense.co.za → About → NumberSense in the Classroom→NumberSense Workbooks

NumberSense Curriculum

The NumberSense Mathematics Programme curriculum is aligned to the South African National Curriculum Statement.

NumberSense.co.za → Resources → Teaching, Planning & Curriculum

CopySafe Projection Files

To enable the projection of the NumberSense Workbook page or exercise that the class is busy with using a data projector, projection files of the NumberSense Workbooks are available for download free of charge.

NumberSense.co.za → Shop → Classroom Resources → Projection Download Grade R-7

NumberSense Mathematics Programme Activity Cards

Download the Activity Cards in English and Afrikaans. The Mosaic Puzzle pieces template is also available for download.

NumberSense.co.za → Resources → Teaching Resources → Geometry (select the applicable phase)

Manipulating Number Teacher Guides and Videos

Manipulating Number (mental mathematics) resources that support the development of reasoning-based calculating strategies in both the Foundation and Intermediate Phase. There are 25 manipulating number questions per page (delivered orally), intended for daily practice.

NumberSense.co.za → Resources → Manipulating Numbers → Mental Mathematics

Teacher Guides

A Comprehensive NumberSense Teacher Guide that brings together all the teacher (and parent) support materials available on the website should be finalised in the next half of the year. As the first draft of the chapters are published, they will be available for download.

NumberSense.co.za → Resources → Teacher Guide

Useful templates

Posters, templates, and resources for class activities.

NumberSense.co.za → Resources → Teaching Resources → Counting and Manipulating Numbers Resources

Assessment

Tasks to support the various roles of assessment: diagnostic, formative, summative.

NumberSense.co.za → Resources → Assessment

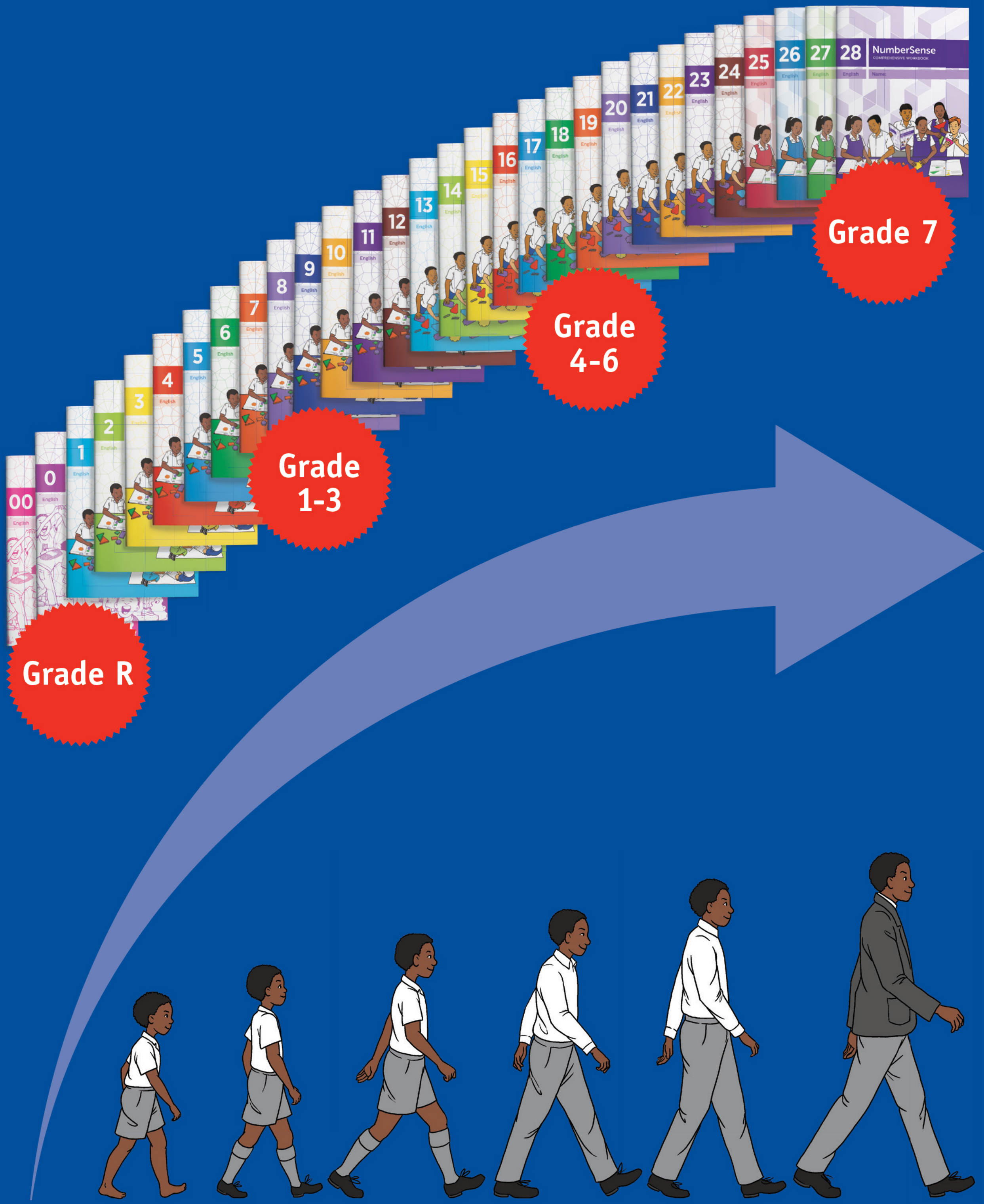
NumberSense support (workshops and conferences)

Detail and access to registration for our upcoming events. NumberSense.co.za → Events

NumberSense Product Guide

Download the latest product guide (price list) & order form. NumberSense.co.za → Shop → Product Guide & Order Form

NUMBERSENSE COMPREHENSIVE WORKBOOK JOURNEY



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